Communicating with a Team of Experts*

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Abstract

This paper combines theories of communication with theories of expertise and teams. Facing a team of experts, who must be given incentives to acquire information and to communicate it truthfully, how can and how should the leader communicate with the team members? We characterize all the possibilities of using the information generated efficiently, and provide a complete welfare ranking of all equilibria. The welfare ranking is shown to depend one for one on the structure of the cost of information acquisition. We discuss applications to task assignment, and to noisy and costly communication.

Keywords: Information Acquisition, Communication, Cheap Talk, Multiple Agents, Expertise

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1 Introduction

1.1 Motivation

Committees have become an integral part of political and corporate decision-making. To name but a few examples, Parliament takes political decisions on behalf of society, the Federal Open Market Committee

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and the Governing Council of the European Central Bank, respectively, decides on monetary policy in the US and Europe, respectively; jurors form a committee that decides on verdicts, and the board of directors of a company is a committee that decides on aspects of the business policies of that company. Many of the decisions taken in these examples are complex, and committee members have to invest time and effort prior to meeting into their understanding of the issues. Once understood the individual opinions have to be communicated and aggregated to enable the committee to reach a well informed decision. Given the importance of committees, it is crucial to understand the nature of interaction among committee members and the consequences of these interactions for the quality of the decisions reached by the committee. These are the questions we address in this paper.

Our investigation is - at least in part - inspired by casual observations of committee decision-making in Universities. All the members being professors, one could argue that they are to a first approximation identical, at least in the ex ante sense. Why then is it that the discussions are often led so asymmetrically? Some professors speak all the time, others are completely silent. Similarly, some members of parliament speak much longer than others. Perhaps more importantly, their fellow members seem to listen more to them than to others. At times, however, discussions end up more democratic, where all the members of the committee speak about the same amount of time. Why?

We develop a simple model that explains when a discussion should be asymmetric and when symmetric. We consider a committee that takes a continuous choice, e.g., the size of a budget. The committee members all agree on the optimal size of the budget. But the problem is that the optimal budget depends on random factors they are only imperfectly informed about and acquiring information about these factors comes at a private cost. Moreover, the committee members cannot observe how much effort their fellow members have spent on information acquisition, nor can they tell whether their statements are true or false. Assuming quadratic loss functions for the payoffs and that prior and acquired information are Normal random variates, the efficient size of the budget is simply an average of prior information and the information that the members observe. Each member's information should receive a weight corresponding to the relative precision of his information. We suppose that the committee cannot commit to the decisions it will take in advance but rather takes these decisions ex post. This assumption seems particularly appealing given there are no conflicts of interests ex post. In the parliament example, e.g., it would be extremely difficult to take a decision that is known to be inefficient by the public. The questions are then, which aggregation rules and information acquisition decisions form an equilibrium and which equilibrium is the best one?

Clearly, our game has a plethora of equilibria as almost any game of cheap talk has. However, viewing the committee as a team in the sense of team theory à la Marshak and Radner [18], it is natural to focus on equilibria that are efficient in a certain sense. We study the class of equilibria that are efficient ex post in the sense that all the information that has been acquired is also transmitted and used.

At first sight one might think that almost anything goes. If a member conjectures that others will listen to him more than to others, then he should acquire more information than others. Vice versa if someone is better informed than the others, then the others should pay relatively more attention to the arguments of this particular member. Surprisingly though, all the equilibria have the same structure. They partition all committee members into two classes, an influential and a negligible one. The negligible agents do not acquire information and nobody pays attention to them. However, the equilibrium is necessarily symmetric among the influential agents. Although information is potentially of a continuous quality, all the influential agents acquire the same amount of information and their opinion has the same impact on the decision made. The reason is essentially that information is a public good in the present model, so that the marginal value of the information that is used in equilibrium is necessarily the same for all agents. As a result, the marginal costs and the effort levels themselves must be equalized in equilibrium as well.

The number of influential committee members has two effects. First, it determines the equilibrium incentives for information acquisition and second, it determines the distribution of costs of information acquisition on the committee-members. If and only if the costs of information acquisition are convex then the equilibria are better, in the sense of a higher joint utility of the team, the more influential members there are. Adding another influential member reduces the fellow influential members' equilibrium effort for information acquisition. However, it also adds another source of information. Moreover, the costs of information acquisition are distributed more evenly in the group. When the cost of information acquisition is convex, then the beneficial effect of adding another source of information outweighs the detrimental effect on members' incentives. As a result, the quality of the decision improves. In addition, the allocation of the cost moves towards a more efficient solution. Both the effect on the aggregate quality of information and the distribution of costs are exactly reversed when there are concave costs of information acquisition.

So, according to our model, discussions should be asymmetric to the maximal extent when the problem is such that there are increasing returns, and discussions should be democratic to the maximal extent when there are decreasing returns in information acquisition.

We extend this simple model into three directions. First, we consider the case of a committee that is formed to reach many decisions, e.g., as many as decisions as there are members. How should the committee allocate the responsibility for the various decisions to its members? Should it have one member responsible per task or should all of them share the responsibility for all the tasks? Specialized responsibility creates better incentives for information acquisition, but it does not exploit the benefits of sharing the costs of information acquisition efficiently. Surprisingly, however, these effects offset each other exactly, rendering both allocations of responsibility welfare equivalent in our model.

Second, we investigate the case of limited communication capacities among members. At times there are sharply decreasing returns to communication, say because communicating is time consuming and time is scarce. How can and how should the president communicate with the members of the committee when she faces a time-capacity limit? She could treat all the members symmetrically, communicating with a certain probability with each one of them; or she could tell some members that she will not be able to talk to them at all and others that they will receive her full attention. When there are decreasing returns to information acquisition, we show that the latter option is preferable among all feasible, asymmetric possibilities. Experts who know they influence the decision for sure have better incentives to acquire information than maybe influential experts. As a result, creating maybe influential experts decreases the aggregate quality of information and the decision worsens. Although the distribution of costs of information acquisition improves, this advantage is insufficient to outweigh the loss of information effect.

Third and finally, we extend our analysis to the case of heterogeneous agents, who differ in their marginal costs of information acquisition. In the interesting case of decreasing returns, the efficient equilibrium now displays a hierarchy. The agent's are, in equilibrium, the more important for the decision the smaller is their marginal cost of information acquisition. When we introduce in addition the realistic feature that communication is noisy, and information is aggregated by a president, we can endogenize the role of this president. Making an agent the president guarantees that her information is used with its full precision, others' information is in part destroyed through the noisy communication channel. It is most efficient to elect the agent with the smallest marginal cost to be the president. Since there is always too little information acquired in equilibrium, the group's objective is essentially to maximize the quality of the decision made. As a result, it is of crucial importance to provide the efficient collectors of information with high incentives. In this sense, our model explains why the best should become president.

1.2 Literature

We combine three lines of thought: communication, endogenous information, and teams. Crawford and Sobel [7] analyze communication between a decision-maker and a single expert. They characterize all equilibrium constellations of communication and show that complete transmission of all information is impossible unless the expert and the decision-maker have identical preferences. The equilibrium with the greatest amount of information transmission Pareto dominates all other equilibria. There are five essential differences with respect to their work. First, all agents have a common ex post objective (that is, for given information) rather than differing preferences. Second, there are multiple senders rather than one. Third, the senders of information have noisy information. Fourth, information is endogenous and its acquisition is subject to a problem of moral hazard. Fifth and finally, the receiver of information observes information on her own.

Szalay [23] studies a model where a decision-maker hires a single expert to acquire information and to provide policy advice. While the advisor is unbiased ex post, he is biased ex ante in the sense that he dislikes the fact that he has to spend costly effort to gather information. The paper characterizes the optimal form of decision-making guaranteeing both an adequate quality of information and the truthful transmission thereof. As in the present work, there are no conflicts of interest ex post, but the only conflicts are ex ante with respect to the costly amount of information acquired. But unlike the present paper, Szalay [23] assumes that the principal can commit in advance to the decisions she takes. Moreover, there are many rather than only one expert in the present investigation.[1]

[1] For a model of information acquisition with conflicts of interests ex post, see Aghion and Tirole [1].

Combining the two approaches of Crawford and Sobel [7] and Szalay [23] in the context of many experts gives rise to a classical team problem in the sense of Marshak and Radner [18]. All the members of the team have a common objective once the cost of information acquisition has been spent. However, the members cannot observe the effort taken by their fellow members, which gives rise to problems of moral hazard. Moreover, members cannot falsify what other members say. Applying the team perspective, we ask what possibilities there are to aggregate the information efficiently and what implications the different forms of information aggregation have on the members' equilibrium incentives for information acquisition.

Various approaches in the literature contain elements of our story, but none of them has them all. Communication with multiple senders has also been analyzed by Austen-Smith [3], Battaglini [4, 5], Krishna and Morgan [15], Ottaviani and Sorensen [21], and Wolinsky [25]. These papers study the ability of the receiver of information to elicit as much as possible information from multiple senders, focussing on the role of the timing of communication, noise in information, or size and direction of senders biases. However, in all these papers the information of the senders is given. In contrast, our paper endogenizes the information acquisition decisions prior to communicating.

Persico [22] and Mukhopadhaya [20] study the role of committee size for incentives for information acquisition. The crucial differences to the current approach are that information acquisition is a binary (rather than a continuous) choice and that information is aggregated through voting (rather than communication). Both Persico and Mukhopadhaya find that large committees face severe problems when it comes to providing their members with incentives to spend the cost of information acquisition. The more members a committee has, the less likely it is that any individual is pivotal. Therefore, smaller committees can make better decisions than larger ones. Our analysis highlights the importance of the fixed costs of information acquisition for these results. Assuming a convex cost of information acquisition where useless information is costless both in total and at the margin, we show that larger committees always make better decisions. Although the free-rider problem is aggravated in a larger committee, each member still acquires some information, and the aggregate quality of information improves. Martinelli [19] shares this feature, although his result is obtained in a model with information aggregation through majority voting.[2]

[2] For models that combine (prevote) communication with voting, see Doraszelski et al. [11] and Vissers and Swank [24].

A model of cheap talk with biased senders, noisy information and communication is studied by Cai [6]. He shows that, surprisingly, preference heterogeneity can provide additional incentives for information acquisition. In contrast to the present approach, he assumes that information acquisition is a binary choice. Moreover, the bias in preferences forces him to consider a particular equilibrium rather than a class. Focussing on the case of common ex post interests, we are able to characterize the complete class of ex post efficient equilibria. Such a description has not been feasible in previous approaches featuring noisy information and conflicts of interests (Battaglini [5] and Cai [6]).[3]

A number of papers take mechanism design approaches to committee problems. Li [16] shows that a committee may benefit from biasing its choice against its prior or preference in order to improve the members' incentives to acquire information. Gerardi and Yariv [13] analyze the complete set of mechanisms available in a binary choice problem and characterize the properties of optimal mechanisms.[4] In contrast to these approaches we do not rely on commitment to choice rules. In many situations with common interests, members of a board have strong incentives to choose the actions that are ex post optimal. So, a commitment to taking inefficient choices, as in Li [16] and Gerardi and Yariv [13] is sometimes hard to justify.

Communication is typically assumed costless in this literature. Exceptions, outside the committee literature are Dessein and Santos [10], who analyze the trade-off between centralized and delegated decision-making when there are gains from costly communication due to a need to coordinate. Dewatripont and Tirole [9] study communication when both the sending (or acquisition) and the receiving of information is costly. Our extension to costly communication explores the consequences of these costs for the organization of committees.

The remainder of the paper is structured as follows. In section two, we introduce the model, section three contains an analysis of the communication game with complete information. In section four, we extend our analysis to the case of incomplete information. In section five we characterize the welfare properties of our equilibria. Section six discusses the extension to the case of multiple tasks, section seven the extension

[3] See also [17] for a model of cheap talk in committees with exogenous information, showing that the optimal form of communication can be implemented through voting.

[4] For a general mechanism design approach to the aggregation of exogeneously given information, see Gerardi et al. [12]. For a model of cheap talk in committees, see Li et al. [17].

A typical feature of committee models is the absence of transfers. For an exception in this respect, see Gromb and Martimore [14]. In their approach, the number of experts influences the informational rents these experts obtain.

to constrained communication, and section eight analyzes noisy communication among heterogenous agents. The final section concludes; longer proofs are relegated to the appendix.

2 The Model

A group of individuals $i = 1, ..., N$ (the agents, henceforth) has to take a decision $x \in \Re$. But the agents are uncertain what decision is best for them. The utility of agent i from taking a decision x depends also on an unknown state of the world, y, according to the common function

$$u_i(x, y) = u(x, y) = -(x - y)^2$$

The agents have a common prior that y is normally distributed with mean μ and precision h (i.e. with variance h^{-1}). Before they take the decision x the individuals can gather information about y. All agents have access to the same fact finding technology, where each agent observes a noisy signal s_i that is informative about the state of the world

$$s_i = y + \varepsilon_i.$$

The noisy component ε_i is independent of y and follows a normal distribution with mean zero and precision e_i. ε_i is independent of ε_j for $i \neq j$, so that the signals are conditionally independent, i.e., $s_i | y$ is independent of $s_j | y$ for $i \neq j$.[5] The cost of finding facts of precision e_i is $c(e_i)$. We assume that useless information is costless, i.e., that $c(\cdot)$ satisfies $c(0) = 0$. Moreover, $c(\cdot)$ is twice differentiable. Additional assumptions on $c(\cdot)$ will be introduced when needed. Agents value taking a decision x and acquiring information of precision e_i according to

$$U_i(x, y, e_i) = u(x, y) - c(e_i).$$

The precision of agents' information (e_i) is not observable to other agents and the facts agents find (s_i) are not verifiable by other agents. Henceforth, we will use precision and effort interchangeably.

Prior to acquiring information, the agents elect a president who will collect the individual pieces of

[5] Independence is analytically convenient but not crucial for our results.

information and aggregate them into a collective choice. Since all agents are ex ante identical, the precise form of information aggregation (that is, through a president) does not really matter, nor does the choice of president itself. For convenience, we order agents the way that agent N is the president. Then, all agents choose the precision of their information. They communicate with the president about what they have found. Finally, the president decides on what decision x to take. Notice, that there are no transfers and the president cannot commit to the actions she takes. This assumption captures the fact that a commitment to inefficient actions is particularly difficult to sustain when all committee members have identical interests ex post. Thus, communication is of the cheap talk form à la Crawford and Sobel (1982).

3 A Benchmark: Communication with Verifiable Information and Observable Precisions

Before we analyze our problem in detail we provide its solution for the case where all the players have symmetric information about the information that each agent has received and where the president can observe the precisions of all agents' signals. Let $\mathbf{s}$ denote the N-vector of signals, $\mathbf{S} = \Re^N$ the set of all realizations of $\mathbf{s}$, and $E_i = \Re_+ \cup 0 \ \forall i$ the set of feasible precisions, and $\mathbf{E} = \times_{i=1}^N E_i$. The strategy of agent i is an effort choice $e_i \in E_i$ for all i. The president chooses in addition a choice rule $x : \mathbf{S} \times \mathbf{E} \rightarrow \Re$. The precisions are chosen non-cooperatively, the choice rule is required to be subgame perfect. We solve our game by backward induction.

3.1 Optimal Decisions

If the president knows the realizations of all N signals s_i and all N effort choices e_i she will choose the action that solves

$$\max_x \left\{ \int_{-\infty}^{\infty} u(x, y) f(y | \mathbf{s}; \mathbf{e}) \, dy \right\}$$

where $f(y | \mathbf{s}; \mathbf{e}) \equiv f(y | s_1, ..., s_N; e_1, ..., e_N)$ is the posterior density of y given all information available. The optimal action is

$$x^* (\mathbf{s}, \mathbf{e}) = E[y | \mathbf{s}; \mathbf{e}]$$

The president's expected conditional indirect utility from choosing $x^*(\mathbf{s},\mathbf{e})$ is

$$Eu\left(x^*(\mathbf{s},\mathbf{e}),y\right) = -E\left[\left(y - E\left[y|\,\mathbf{s};\mathbf{e}\right]\right)^2\Big|\,\mathbf{s};\mathbf{e}\right] = -Var\left[y|\,\mathbf{s};\mathbf{e}\right]$$

The optimal choice and the indirect expected utility correspond to the first and the second moment of the posterior distribution, respectively. It is well known that the moments of the posterior are given by (see, for example, deGroot [8])

$$E\left[y|\,\mathbf{s};\mathbf{e}\right] = \frac{h}{h+A(\mathbf{e})}\mu + \sum_{i=1}^{N}\frac{e_i}{h+A(\mathbf{e})}s_i \tag{1}$$

and

$$Var[y\,|\mathbf{s};\mathbf{e}] = \frac{1}{h+A(\mathbf{e})} \tag{2}$$

where $A(\mathbf{e}) \equiv \sum_{i=1}^{N} e_i$ is the sum of all signal precisions. Observe that the conditional variance is independent of $\mathbf{s}$, a convenient property of normal distributions. Thus, the ex ante expected utility of agent i is

$$E\left[U_i\left(x^*(\mathbf{s},\mathbf{e}),y,e_i\right);\mathbf{e}\right] = -\frac{1}{h+A(\mathbf{e})} - c(e_i)\,.$$

3.2 Optimal Precisions

Agents choose the precision of their information non-cooperatively. The optimal choice of precision of agent i, $e_i^*(\mathbf{e}_{-i})$ [6] solves

$$\max_{e_i}\left\{-\frac{1}{h+A(\mathbf{e})} - c(e_i)\right\}$$

We assume that the marginal cost of effort is either monotone increasing, monotone decreasing, or constant. For ease of exposition, we discuss these cases in sequence.

[6] Troughout the paper we omit arguments of functions that are constant throughout the analysis and write $e_i^*(\mathbf{e}_{-i})$ for $e_i^*(\mathbf{e}_{-i},h)$.

3.2.1 The Case of Strictly Convex Costs

In this case it is convenient to assume that useless information is also costless at the margin, i.e., $c'(0) = 0$. As a result, the positive marginal value of information exceeds the zero marginal cost for the first unit of information. Moreover, since this problem is strictly concave in e_i, $e_i^*(\mathbf{e}_{-i})$ exists and is unique for each $\mathbf{e}_{-i}$. $e_i^*(\mathbf{e}_{-i})$ satisfies the first-order condition

$$\frac{1}{\left(h + A\left(\mathbf{e}_{-i}, e_i^*\left(\mathbf{e}_{-i}\right)\right)\right)^2} = c'\left(e_i^*\left(\mathbf{e}_{-i}\right)\right). \tag{3}$$

At the optimum, the marginal value of information must match the marginal cost of increasing the precision of information. Notice that the marginal value of information depends only on $A(\mathbf{e})$. As a result, the marginal costs of information acquisition must in equilibrium be equalized for all i. Since the marginal cost is monotonic in e the precision levels themselves must be equalized in equilibrium. It follows that any equilibrium must satisfy the condition

$$\frac{1}{\left(h + Ne^*\right)^2} = c'\left(e^*\right). \tag{4}$$

It is easy to see that there exists exactly one solution of (4). So the unique subgame perfect equilibrium of the game is $e_i = e^*$ for all i and the corresponding choice function $x^*(\mathbf{s}, \mathbf{e}^*)$.

3.2.2 The Case of Concave Costs

It is possible to sustain the equilibrium defined by (4) also when costs are concave, under certain conditions. We prove the following result in the appendix:

Lemma 1 *Let the cost function satisfy* $-2\left(c'(e)\right)^{\frac{3}{2}} - c''(e) < 0$ *for all* e *and suppose in addition that* $c'(0) < h^{-2}$. *Then,* (4) *has a unique solution for* $N = 1$, *say* $\underline{e}$. *If* $c'(0) < \left(h + \underline{e}\right)^{-2}$ *then the game has a unique subgame perfect equilibrium in which* $e_i = e^*$ *for all* i, *where* e^* *is the unique solution to (4). If* $c'(0) \geq \left(h + \underline{e}\right)^{-2}$ *there are* N *equilibria. In any of these, exactly one agent exerts effort* $e_i^* = \underline{e}$, *and the remaining agents exert* $e_j^* = 0$.

Obviously, when the costs are very concave, only trivial equilibria arise. However, with mildly increasing returns, the symmetric equilibrium still exists. To see the role of our assumptions, compute the second derivative of agent $i's$ payoff function with respect to e_i

$$\frac{-2}{\left(h + A\left(\mathbf{e}_{-i}, e_i\right)\right)^3} - c''\left(e_i\right) \tag{5}$$

Suppose that the marginal cost of effort is "sufficiently" small (see the appendix for a precise statement) and suppose there is a stationary point, say $e_i^*\left(\mathbf{e}_{-i}\right)$, that satisfies (3). Evaluating (5) around $e_i^*\left(\mathbf{e}_{-i}\right)$ and using the restriction on the degree of concavity, we get $\frac{-2}{\left(h+A\left(\mathbf{e}_{-i}, e_i^*\left(\mathbf{e}_{-i}\right)\right)\right)^3} - c''\left(e_i^*\left(\mathbf{e}_{-i}\right)\right) = -2\left(c'\left(e_i^*\left(\mathbf{e}_{-i}\right)\right)\right)^{\frac{3}{2}} - c''\left(e_i^*\left(\mathbf{e}_{-i}\right)\right) < 0$. Thus, given the assumption[7], any stationary point must be a maximum. But then, there is at most one such point. We prove the existence of the stationary point, defined by (4) in the appendix. On the other hand, if the marginal cost of effort is high for small effort, then only one agent will acquire information, and there are N possibilities how the agents can coordinate on who acquires the information. These results carry over directly to the case of linear costs of information acquisition, where we can write $c\left(e\right) = c'\left(0\right)e$.

In what follows we will maintain the assumptions on $c\left(\cdot\right)$ introduced in this section. Moreover, we assume that $c'\left(0\right)$ is sufficiently small to rule out the coordination equilibria mentioned in lemma 1, where only one agent acquires information.

3.3 Inefficiency of the Equilibrium

For the purpose of welfare comparisons we need some possibility to exchange utility between agents. One possibility to achieve this is to allow the agents to exchange transfers $T_i\left(\cdot\right)$ and endow them with a utility function of the form

$$U_i\left(x, y, e_i, T_i\right) = u\left(x, y\right) - c\left(e_i\right) + v\left(T_i\left(\cdot\right)\right)$$

where $v\left(\cdot\right)$ displays infinite risk aversion, i.e., $Ev\left(T_i\right)$ is equal to the minimum transfer in the support of T_i. When transfers are feasible, then the president can use them ex ante to compensate the agents for costs of

[7] An example that satisfies the assumption is $c\left(e\right) = \ln\left(1+e\right)$.

information acquisition. However, it is not optimal to make the transfers contingent on the outcome. (For other models using this approach, see Aghion and Tirole (1997) and Szalay (2005).) Using budget balance for the transfers, the socially optimal precision levels are the solution of the problem

$$\max_{e_1,\ldots,e_N} \left\{ -\frac{N}{h + A(\mathbf{e})} - \sum_{i=1}^{N} c(e_i) \right\}$$

In case the cost is convex, the solution is symmetric among agents and satisfies the first-order condition

$$\frac{N}{(h + Ne^{**})^2} = c'(e^{**}) \tag{6}$$

At the optimum, all agents marginal costs of acquiring information are equal to the social marginal value of information. We observe that the Nash-equilibrium is inefficient, i.e., that $e^* < e^{**}$. The reason is that agents neglect the fact that information has public value and choose a level of precision that does not reflect the external effects on their fellow agents.

Similarly, in case the cost is concave and the marginal cost at zero effort is sufficiently small, the solution involves only one agent acquiring information of a precision that satisfies

$$\frac{N}{(h + e^{**})^2} = c'(e^{**})$$

In this case the Nash-equilibrium may be inefficient for two reasons. First, because too many agents acquire information, and second, because each one of them neglects the external effects of his information acquisition on the utility of others. Finally, in case of linear costs, the allocation of information acquisition among agents is irrelevant; only the aggregate precision is determined at the optimum, and the Nash-equilibrium is inefficient in the sense that the aggregate precision of information is inefficiently low.

4 Communication with Non-observable Precisions

We now address our game when precisions are not observable to others and information is not verifiable by others. Formally, we face a model of cheap talk. Such models generically have a plethora of equilibria and ours is no exception in this respect. We will focus on a particular class of equilibria of our game and investigate whether equilibria within this class exist and analyze their properties. In particular, we investigate whether our model has equilibria where all the information that has been acquired is truthfully transmitted. We term these equilibria fully informative. These equilibria are appealing because they Pareto dominate all other equilibria ex post, so agents have strong interests to coordinate on playing the strategies associated with these equilibria.

We begin by defining agents' strategies in this enriched game of incomplete information and the notion of a fully informative equilibrium.

4.1 Fully Informative Equilibria

To take account of non-verifiable information we allow agents to send messages to the president and let the equilibrium behavior of agents and president determine the precise meaning of these messages. We let $M_i = \Re$ denote the message space of member i with generic element m_i, and $\mathbf{M} = \times_{i=1}^{N-1} M_i$ the product set of the message spaces with generic element $\mathbf{m}$, an $N-1$-vector of messages. As before $\mathbf{s}$ and $\mathbf{e}$ denote the vectors of signals and their precisions, $\mathbf{s}_{-i}$ and $\mathbf{e}_{-i}$ denote these vectors after removing s_i and e_i. $S_i = \Re$ is the set of feasible signal realizations of agent $i's$ signal, and E_i denotes the feasible precisions of agent i's information. A strategy for agent $i = 1, ..., N-1$ (all but the president) is a choice of precision $e_i \in E_i$ and a reporting strategy $m_i : S_i \times E_i \to \Re$. The president's strategy is an effort choice $e_N \in E_N$ and a decision rule $x : S_N \times E_N \times \mathbf{M} \to \Re$. Our equilibrium concept is Perfect Bayes-Nash equilibrium. So the agents' and the president's beliefs are computed from Bayes law and the agents' strategies.

Heuristically, a fully informative equilibrium is a situation where the president conjectures some constellation of precisions for all agents, chooses an optimal level of precision herself, and takes all the messages she receives at face value. So she believes that her posterior is the true posterior given all the information available in the group and she takes a corresponding action which is optimal given that information. On the other hand all agents find it optimal to acquire information of the conjectured precision and report their findings truthfully to the president.

Formally, we define a fully informative equilibrium as a set of strategies and beliefs that satisfy the following requirements:

1) The president takes every agent's advice at face value and believes that $m_i = s_i$ for all s_i and all i, and she conjectures that agent i acquires information of some precision e_i for all i. As a result, her posterior belief of y given her signal realization and the messages she received is $f(y|\, s_N, \mathbf{m}, \mathbf{e})$, the true posterior density when $\tilde{\mathbf{s}}_{-N} = \mathbf{m}$.

2) The president's optimal choice of action, $x^*(\mathbf{m}, s_N, \mathbf{e})$ solves

$$\max_x \left\{ \int_{-\infty}^{\infty} u(x,y)\, f(y|\, s_N, \mathbf{m}, \mathbf{e})\, dy \right\} \tag{7}$$

The solution is the affine function

$$x^*(\mathbf{m}, s_N, \mathbf{e}) = (1 - \sum_{i=1}^{N} \alpha_i^*(\mathbf{e}))\mu + \sum_{i=1}^{N-1} (\alpha_i^*(\mathbf{e})\, m_i) + \alpha_N^*(\mathbf{e})\, s_N \tag{8}$$

where

$$\alpha_i^*(\mathbf{e}) = \frac{e_i}{h + A(\mathbf{e})} \forall i \tag{9}$$

$\alpha_i^*(\mathbf{e})$ corresponds to the statistically optimal weight assigned to the information of agent i in (1) for the conjectured precision choices $\mathbf{e}$.

3) In equilibrium, the president's conjectured precisions for all agents must be the correct ones and indeed all agents must send truthful messages. As a result, the decision taken by the principal corresponds to the ex post efficient choice given all the available information, and her unconditional expected utility gross of information acquisition costs is given by (2). Thus, her optimal effort choice $e_N^*(\mathbf{e}_{-N})$ solves

$$\max_{e_N} \left\{ -\left(h + \sum_{i=1}^{N-1} e_i + e_N \right)^{-1} - c(e_N) \right\}. \tag{10}$$

4) The optimal reporting strategy of agent i maximizes agent $i's$ expected utility taking account of the president's choice rule (8) and the president's belief that the agent reports truthfully. Moreover, in a fully informative equilibrium, agent i believes that all other agents send fully informative messages. So agent i identifies the random variables $\tilde{\mathbf{m}}_{-i}$ with $\tilde{\mathbf{s}}_{-i,N}$ as well as their realizations in (8). Formally, let

$f\left(y,\mathbf{s}_{-i}|\, s_i,\mathbf{e}\right)$ denote the conditional likelihood that $\tilde{y}=y$ and $\tilde{\mathbf{s}}_{-i}=\mathbf{s}_{-i}$ given $\tilde{s}_i=s_i$ and given $\mathbf{e}$. Then, the optimal reporting strategy $m_i^*\left(s_i,\mathbf{e}_{-i},e_i\right)$, solves[8]

$$\max_{m_i}\left\{\int_{-\infty}^{\infty}\int\cdots\int_{-\infty}^{\infty} u\left(x^*\left(\mathbf{s}_{-i},m_i,\mathbf{e}\right),y\right) f\left(y,\mathbf{s}_{-i}|\, s_i,\mathbf{e}\right) dyds_1\cdots ds_N\right\}. \tag{11}$$

5) Let $f\left(y,\mathbf{s}|\,\mathbf{e}\right)$ denote the unconditional joint density of y and $\mathbf{s}$. Then, agent $i's$ optimal precision choice $e_i^*\left(\mathbf{e}_{-i}\right)$ solves

$$\max_{\hat{e}_i}\left\{\int_{-\infty}^{\infty}\int\cdots\int_{-\infty}^{\infty} u\left(x^*\left(\mathbf{s}_{-i},m_i^*\left(s_i,\mathbf{e},\hat{e}_i\right),\mathbf{e}\right),y\right) f\left(y,\mathbf{s}|\,\mathbf{e}_{-i},\hat{e}_i\right) dyds_1\cdots ds_N - c\left(\hat{e}_i\right)\right\}. \tag{12}$$

Requirements 1) through 5) are consistent if and only if the solution to (12) satisfies $e_i=e_i^*\left(\mathbf{e}_{-i}\right)$ for all i and the solution to (11) satisfies $m_i^*\left(s_i,\mathbf{e}^*\right)=s_i$ for all s_i and all i.

Although the definition of equilibrium is somewhat involved, there is an intuitive way to solve the game. We will split our analysis into two parts. First, we analyze the case where agents cannot lie, say because their information is verifiable. Formally, we constrain agents to send messages $m_i\equiv s_i$ and characterize the complete set of equilibria of this simplified game. In a second step we allow agents to choose any reporting strategy, that is, we go back to the case of non-verifiable information. By definition, the set of fully informative equilibria must be a subset of the set of equilibria of the game with verifiable information. Formally, non-verifiability acts like a constraint on the equilibrium choices that ensures that truthtelling is optimal on equilibrium path.

4.2 The Case of Verifiable Information

There is no need to distinguish messages and signals when information is verifiable, so for the analysis in this section we have $m_i\equiv s_i$. Moreover, for all agents $i=1,...,N-1$ the strategy is reduced to a choice of precision. The president's strategy is unchanged, but her belief is identified with the true posterior distribution given her choice of precision and a conjecture for the remaining precisions $\mathbf{e}_{-N}$.

We now proceed to characterize the best responses of agents and the president, vice versa, to the actions

[8] Srictly speaking the formulation is true for $i\neq 1$. For $i=1$ the obvious changes apply.

taken by the others. We already know the optimal way to weight the information provided by agents. By (8), the optimal decision is an affine function of the signals, where the weight attached to each signal is given by (9). Viewed as a function of all precision levels, the president's best response with respect to aggregating information is to weight the information according to

$$\alpha_i(\mathbf{e}) \equiv \frac{e_i}{h + A(\mathbf{e})}. \tag{13}$$

It follows that, if there is an equilibrium, then the decision taken by the president will be a point on an affine function. As a result, we can limit attention to agents' best responses to linear aggregation rules. So let $\boldsymbol{\alpha} \in \boldsymbol{\Delta}^N$ denote a vector of weights. $\boldsymbol{\Delta}^N$ is the N-simplex and α_0 is the weight attached to the prior. With a slight abuse of notation, we let $x(\boldsymbol{\alpha}, \mathbf{s}, e_N)$ denote the president's decision function. Define the unconditional expected utility as $V(\boldsymbol{\alpha}, \mathbf{e}) \equiv E[u(x(\boldsymbol{\alpha}, \mathbf{s}, e_N), y); \mathbf{e}]$. For a given $\mathbf{e}$ and $\boldsymbol{\alpha}$, we have

$$\begin{aligned} V(\boldsymbol{\alpha}, \mathbf{e}) &\equiv -\int_{-\infty}^{\infty} \int \cdots \int_{-\infty}^{\infty} \left(\mu - y + \sum_{j=1}^{N} \alpha_j (s_j - \mu) \right)^2 f(\mathbf{s}, y; \mathbf{e})\, ds_1 \cdots ds_N dy \\ &= -\left(Var(y) + \sum_{j=1}^{N} \alpha_j^2 Var(s_j) + 2\sum_{j=1}^{N} \sum_{k=j+1}^{N} \alpha_j \alpha_k Cov(s_j, s_k) - 2\sum_{j=1}^{N} \alpha_j Cov(s_j, y) \right) \end{aligned}$$

where the second line follows from squaring and integrating out. Our informational setup implies that $Var(y) = Cov(s_j, s_i) = Cov(s_i, y) = h^{-1}$ for $i, j = 1, ..., N$ and $i \neq j$ and $Var(s_i) = \left(h^{-1} + e_i^{-1}\right)$. So, we can write

$$V(\boldsymbol{\alpha}, \mathbf{e}) = -\left(h^{-1} + \sum_{j=1}^{N} \alpha_j^2 \left(h^{-1} + e_j^{-1}\right) + 2\sum_{j=1}^{N-1} \sum_{k=j+1}^{N} \alpha_j \alpha_k h^{-1} - 2\sum_{j=1}^{N} \alpha_j h^{-1} \right)$$

The best reply of agent i to the president's aggregation rule and all fellow agents' precision levels solves

$$\max_{e_i} \{V(\boldsymbol{\alpha}, \mathbf{e}) - c(e_i)\}$$

$V(\boldsymbol{\alpha}, \mathbf{e})$ is concave in e_i and satisfies $\lim_{e_i \to 0} \frac{\partial}{\partial e_i} V(\boldsymbol{\alpha}, \mathbf{e}) = \infty(0)$ for $\alpha_i > (=)\, 0$. For $\alpha_i = 0$, the solution is

$e_i^*(\alpha_i) = 0$. For $\alpha_i > 0$, the solution must satisfy the first-order condition

$$\frac{\alpha_i^2}{(e_i^*(\alpha_i))^2} - c'(e_i^*(\alpha_i)) = 0.$$

Clearly, if $c(\cdot)$ is convex, then there is a unique solution to the first-order condition. In case $c(\cdot)$ is concave, we assume that it satisfies the condition $-2c'(e) - ec''(e) < 0$ for all e. By the now familiar argument, this condition implies that the second order condition is satisfied around any stationary point, that is, $\frac{-2\alpha_i^2}{(e_i^*(\alpha_i))^3} - c''(e_i^*(\alpha_i)) = \frac{-2c'(e_i^*(\alpha_i))}{e_i^*(\alpha_i)} - c''(e_i^*(\alpha_i)) < 0$. As a result, there exists a unique stationary point, corresponding to the solution the agent's problem.

Define the function $q(e) \equiv e(c'(e))^{\frac{1}{2}}$. Notice that $q(0) = 0$ because $c'(0) < \infty$. Moreover, $q'(e) = (c'(e))^{\frac{1}{2}} + \frac{1}{2}e(c'(e))^{-\frac{1}{2}}c''(e) > 0$ by the assumption that $-2c'(e) - ec''(e) < 0$. Thus, the function $b(\cdot) \equiv q^{-1}(\cdot)$ exists. Using the first-order condition and the definition of $b(\cdot)$, we can write the best reply of agent i, viewed as a function of α_i, as

$$e_i = b(\alpha_i). \tag{14}$$

By our assumptions on $c(\cdot)$, we have $b(0) = 0$.

If the president's aggregation rule and the agents' precision choices are in equilibrium then the decision taken by the president is ex post efficient. For the case of convex costs, her optimal choice of precision satisfies the first-order condition

$$\frac{1}{\left(h + e_N^*(\mathbf{e}_{-N}) + \sum_{j=1}^{N-1} e_j\right)^2} - c'(e_N^*(\mathbf{e}_{-N})) = 0.$$

For the case of concave costs, the first-order condition describes her optimal choice only for $\sum_{j=1}^{N-1} e_j$ sufficiently small, for $\sum_{j=1}^{N-1} e_j$ large enough, she prefers to choose zero effort. Following lemma 1, it suffices to consider the former case (because in equilibrium the effort of all other agents is sufficiently small), so we can define the president's best reply function as the solution of the first-order condition viewed as a function of $\mathbf{e}_{-N}$

$$e_N = d\left(\mathbf{e}_{-N}\right) \tag{15}$$

An equilibrium is a vector of weights and a vector of precisions, $\{\boldsymbol{\alpha}^*, \mathbf{e}^*\}$, that satisfy (13), (14), and (15).

Proposition 1 *Suppose signals are verifiable and their precisions are not observable. Then, there exist* 2^{N-1} *fully informative equilibria. In each of these equilibria agents are partitioned into two groups. Group one comprises the president and* $0 \leq \overline{N} \leq N-1$ *agents, who all acquire information of the same precision,* $e^*\left(\overline{N}\right)$. $e^*\left(\overline{N}\right)$ *is uniquely determined by the condition*

$$\frac{1}{\left(h + e^*\left(\overline{N}\right)\left(\overline{N}+1\right)\right)^2} = c'\left(e^*\left(\overline{N}\right)\right).$$

The precision of the remaining $N-1-\overline{N}$ *agents is zero. The president gives equal weight to the information of the agents in the first group,* $\alpha^*\left(\overline{N}\right) = \frac{e^*\left(\overline{N}\right)}{h+e^*\left(\overline{N}\right)\left(\overline{N}+1\right)}$, *and attaches a weight of zero to the information of agents in the second group.*

Proof. From (14) and (15), the aggregation rule and agents' $i = 1, ..., N-1$ effort choices are best replies to each other for given e_N iff

$$\alpha_i = \frac{b\left(\alpha_i\right)}{h + e_N + \sum_{j=1}^{N-1} b\left(\alpha_j\right)} \text{ for } i = 1, ..., N-1 \tag{16}$$

Notice that $\alpha_i = 0$ and $b\left(\alpha_i\right) = 0$ satisfies this condition. Suppose that $\alpha_i > 0$ for $i = j, k$. Then, condition (16) implies that

$$\frac{\alpha_j}{b\left(\alpha_j\right)} = \frac{\alpha_k}{b\left(\alpha_k\right)}. \tag{17}$$

But, by the first-order condition of the agent's choice of precision and the definition of $b\left(\cdot\right)$ we have $\alpha_j = b\left(\alpha_j\right)\left(c'\left(b\left(\alpha_j\right)\right)\right)^{\frac{1}{2}}$ and $\alpha_k = b\left(\alpha_k\right)\left(c'\left(b\left(\alpha_k\right)\right)\right)^{\frac{1}{2}}$. It follows that whenever $\alpha_i > 0$ for $i = j, k$ then

$$c'\left(b\left(\alpha_j\right)\right) = c'\left(b\left(\alpha_k\right)\right). \tag{18}$$

Since all the cost functions are identical and the marginal costs monotonic (both when cost is either concave or convex), it follows that the precision levels of all agents whose information has positive weight are equalized, i.e., $e_j = e_k$. Suppose $\overline{N}$ agents are given a positive weight, and let $e_{\overline{N}}$ denote the common level of the precision of their information. Combining (16) and the symmetry condition (18), we obtain

$$\left(c'\left(e_{\overline{N}}\right)\right)^{\frac{1}{2}} = \frac{1}{h + e_N + \overline{N}e_{\overline{N}}}. \tag{19}$$

Consider first the case of convex costs where in addition $c'(0) = 0$. Then, the left-hand side of (19) is an increasing function of $e_{\overline{N}}$ that takes a value of zero at $e_{\overline{N}} = 0$. The right-hand side is a decreasing function of $e_{\overline{N}}$ that takes value $(h + e_N)^{-1}$ at $e_{\overline{N}} = 0$. So, for all $\overline{N}$, h and e_N, (19) has a unique solution, $e_{\overline{N}} = e_{\overline{N}}(e_N)$. Viewed as a function of e_N, the solution can be interpreted as a "best reply" of important agents to the president's precision of information.

The president's best reply to $\overline{N}$ agents choosing precision $e_{\overline{N}}$, $e_N = d\left(\mathbf{e}_{\overline{N}}\right)$, is the solution to the first-order condition

$$c'\left(e_N\right) = \frac{1}{\left(h + e_N + \overline{N}e_{\overline{N}}\right)^2} \tag{20}$$

By the arguments given above, (20) has a unique solution. Since the right-hand side of (20) is simply the squared version of the right-hand side of (19), it follows once again that the precision levels must be equalized, so $e_{\overline{N}} = e_N$. Using symmetry once more, we obtain the condition

$$c'(e) = \frac{1}{\left(h + \left(\overline{N} + 1\right)e\right)^2} \tag{21}$$

which has a unique solution by the arguments given above. Moreover, the solution corresponds to $e^*\left(\overline{N}\right)$ as defined in the proposition. The number of equilibria is computed from all combinations of members who do or do not acquire information. There are $\sum_{i=0}^{N-1} \begin{pmatrix} N-1 \\ N-1-i \end{pmatrix} = 2^{N-1}$ such combinations.

The proof of the case of concave costs follows directly from the proof of lemma 1 with $\overline{N}+1$ replacing N. ■

Any number of agents but the president not acquiring information is consistent with equilibrium. Suppose the president neglects an agent's opinion, that is, $\alpha_i = 0$ for some i. Then, agent i is strictly worse off acquiring costly information rather than none, because his information will not affect the president's decision. On the other hand, if $e_i = 0$ for some i then the best reply of the president is clearly to not consider that agent's information in her decision-making. This reasoning applies for all agents but the president, because the president knows how precise her information is. Therefore, an improvement in her information necessarily improves the decision made, so the marginal value of additional information is always positive for the president and she always acquires information of positive precision.

Not any conjecture about precision levels of the agents is consistent with equilibrium. On the contrary, equilibrium constellations of strictly positive precisions are necessarily symmetric among all positive precisions. In other words, whenever an agent's information has an impact on the president's decision then the magnitude of this impact must be the same as the magnitude of the impact of any other agent's information who influences the decision of the president. More formally, if $\alpha_j, \alpha_k > 0$ for j and k, then the impact on the decision made per unit of precision acquired must be the same for agents j and k, that is $\frac{\alpha_j}{b(\alpha_j)} = \frac{\alpha_k}{b(\alpha_k)}$. But by the definition of an agent's best reply we have $\frac{\alpha_j}{b(\alpha_j)} = \left(c'\left(b\left(\alpha_j\right)\right)\right)^{\frac{1}{2}}$. It follows that the marginal costs of acquiring information must be the same for agents j and k, so their equilibrium precisions must be identical.

Finally, the president acquires the same amount of information as any of the other agents who acquire information in equilibrium (if there are any such agents). In other words, the fact that the president knows the precision of her information but only conjectures the precisions of the informations of other agents does not alter her equilibrium precision. The reason is that the equilibrium marginal value of information is the same for all agents, so the equilibrium marginal costs of information acquisition must be the same as well.

4.3 The Case of Non-verifiable Information

Proposition 1 provides us with a complete description of the entire set of equilibria of our game when information is verifiable. That in itself would not be so interesting, if it were not exactly the same set of constellations that also survive as fully informative equilibria in the original game where information is not verifiable. Moreover, since the consistency of the aggregation rule and the precision choices is a necessary condition for equilibrium of our game, we know that no other constellation can ever be a fully informative equilibrium. Formally, we have the following result.

Proposition 2 *Any equilibrium of the game with verifiable information is also a fully informative equilibrium of the game with non-verifiable information.*

The intuition for this results is as follows. We must rule out any deviation by any agent both with respect to his choice of precision and with respect to his reporting strategy. Deviations in terms of the reporting strategy alone are easy to rule out. Suppose the president conjectures that agent i's information is of precision $e^*\left(\overline{N}\right)$, the equilibrium precision when a total of $\overline{N}+1$ agents acquires information of positive precision. If the agent indeed has information of precision $e^*\left(\overline{N}\right)$ and the president takes the advice of this agent at face value then this agent wants to be truthful. Essentially, this insight corresponds to the observation that cheap talk games have informative equilibria when there is no conflict.

But the agent's strategy space is richer. A particularly appealing deviation for the agent is to choose precision zero and always send the message $m_i = \mu$. By sending this message all the time, the agent ensures that his information enters decision-making the correct way. However, there is a second effect that the agent does not correct this way. If the agent shirks his contribution to information acquisition, then the information provided by those agents who stick to the equilibrium precisions should receive a larger weight, because the precision of their information relative to the aggregate precision has increased. It turns out, that any deviation the agent might consider, will result in a reduction of expected utility for that agent.

We briefly sketch our proof. The details of the argument can be found in the appendix.

Sketch of Proof. Reorder the agents such that $i = 1, ..., \overline{N}+1$ are the relevant ones, that is those who are supposed to have information of positive precision. (The last one is the president.) Let $\mathbf{e}^*$ denote a vector with $\overline{N}+1$ entries $e^*\left(\overline{N}\right)$ and let $\mathbf{e}^*_{-i}$ denote a vector with $\overline{N}$ entries $e^*\left(\overline{N}\right)$. If the agent has information of precision $\hat{e}_i$, then the report that maximizes his conditional expected utility is

$$m_i^*\left(s_i, \mathbf{e}^*, \hat{e}_i\right) = \mu + \frac{\hat{\alpha}_i}{\alpha^*}\left(s_i - \mu\right) - \sum_{j \neq i}^{\overline{N}+1}\left(\frac{\left(\alpha^* - \hat{\alpha}_{-i}\right)}{\alpha^*}\left(E\left[s_j \left| s_i; \mathbf{e}^*_{-i}, \hat{e}_i\right.\right] - \mu\right)\right) \tag{22}$$

where $\alpha^* \equiv \alpha^*(\mathbf{e}^*)$ is the weight the president attaches to the agent's information, and $\hat{\alpha}_i$ and $\hat{\alpha}_{-i}$, respectively, are the statistically optimal weights of the information of agent i and all other agents, respectively, when the former has information of precision $\hat{e}_i$ and the latter have information of precision $e^* \equiv e^*\left(\overline{N}\right)$. If the precision of agent i's information is smaller (larger) than the president conjectures, then $\hat{\alpha}_i < (>)\, \alpha^*$ and $\hat{\alpha}_{-i} > (<)\, \alpha^*$. In words, the president overestimates the precision of agent i's information and underestimates the precisions of all other agents. The optimal report of agent i takes these biases into account and corrects for them to some extent. Consider the first bias and suppose for concreteness that the agent's precision is too small. If the president would listen only to agent i, then agent i would report $m_i^*(s_i, e_i^*, \hat{e}_i) = \mu + \frac{\hat{\alpha}_i}{\alpha^*}(s_i - \mu)$. Since $\frac{\hat{\alpha}_i}{\alpha^*} < 1$, this report is a compromise between the ex ante mean and the true signal realization. By making the report more conservative than the signal, the agent achieves that the decision taken by the principal is optimal given its true precision. Consider next the bias that arises from the presence of other agents that acquire information. Agent i knows that the president attaches too little weight than to their information. Moreover, he uses his signal s_i to infer what signals they might have received. If $s_i > \mu$ then agent i thinks the other agents have also received a signal larger than μ. Since their information receives too little weight, agent i revises his report upwards to ensure that the direction of their information receives more weight. Which effect dominates depends on the magnitude of agent i's deviation. For small deviations, the latter effect dominates when $\overline{N}$ is sufficiently large. For large deviations, the former effect may dominate.

Taking account of this optimal reporting strategy, the agent's ex ante expected utility can be computed as

$$\begin{aligned} &\int_{-\infty}^{\infty}\int \cdots \int_{-\infty}^{\infty} u\left(x^*\left(\mathbf{s}_{-i}, m_i^*\left(s_i, \mathbf{e}_{-i}^*, \hat{e}_i\right), \mathbf{e}^*\right), y\right) f\left(y, \mathbf{s}\middle|\, \mathbf{e}_{-i}^*, \hat{e}_i\right) dy ds_1 \cdots ds_N - c(\hat{e}_i) \qquad (23) \\ = \quad & -\overline{N}\left(\alpha^* - \hat{\alpha}_{-i}\right)^2 \left(\frac{1}{\hat{e}_i + h} + \frac{1}{e^*}\right) - 2\left(\alpha^* - \hat{\alpha}_{-i}\right)^2 \sum_{j=2}^{\overline{N}} \left(\overline{N} + 1 - j\right) \frac{1}{\hat{e}_i + h} - \frac{1}{h + \hat{e}_i + \overline{N} e^*} - c(\hat{e}_i) \end{aligned}$$

The optimal precision of information for agent i maximizes (23) with respect to $\hat{e}_i$.

The first two expressions are equal to zero if and only if $\hat{e}_i = e^*$. For $\hat{e}_i \neq e^*$, both terms are strictly negative. The last two terms correspond to the agent's net payoff when information is used efficiently. We know that $\hat{e}_i = e^*$ is the unique maximizer of these two terms in isolation. Putting both observations together shows that $\hat{e}_i = e^*$ corresponds to the unique optimum of the agent's problem. So the unique optimal strategy of the agent is to acquire information of precision e^* and reveal his signal truthfully. ■

Our result has an important interpretation in terms of preplay communication. Suppose we allow the president to make announcements to individual agents. What announcements are credible in the sense that, if believed by the agents, induce the president to behave in a way that rationalizes the announcement? By propositions one and two, the president can tell individual agents that she will not listen to them or that she will listen to each agent the same way. Nothing else can be rationalized this way. In other words, the president can choose the number of experts and ignorants to have in a team. This observation leads immediately to the question, how many experts a group should have.

5 Efficiency Properties of Informative Equilibria

Suppose the president tells some agents in advance that she will not take their opinions into account. For convenience, suppose she announces to the first $\overline{N}$ agents that they will be experts. Let the president herself be the $\overline{N}+1th$ agent. Let $V\left(\overline{N}\right) \equiv V\left(\boldsymbol{\alpha}^*, \mathbf{e}^*\right)$ denote the expected utility (gross of costs of information acquisition) of an agent when the size of the expert group is $\overline{N}+1$ ($\overline{N}$ agents plus the president) and let

$$W\left(\overline{N}\right) = -NV\left(\overline{N}\right) - \sum_{i=1}^{\overline{N}+1} c\left(e^*\left(\overline{N}\right)\right) \tag{24}$$

denote the expected utility of the entire group when the expert group has size $\overline{N}+1$.

Suppose the president neglects some agents, so $\overline{N} < N-1$. Could the group reach a higher utility if the number of experts were increased by, say one agent? On the one hand, each agent will have different incentives to acquire information, because his impact on the decision made is different. On the other hand, the president will base her decision on a greater number of signals. Finally, the distribution of costs is more equalized across the members. If the costs of information acquisition are convex, then adding one member decreases the equilibrium incentives of each individual member but increases the aggregate quality of information. In addition, the total cost of information acquisition is distributed more efficiently. As a result, welfare increases. If the cost is concave, then adding another member worsens individual equilibrium incentives and decreases the aggregate quality of information. Moreover, it worsens the distribution of total costs. We now make these arguments formal.

The equilibrium choice of precision as a function of the number of experts has the convenient representation

$$\frac{1}{\left(h+\left(\overline{N}+1+\gamma\right)e\left(\gamma\right)\right)^{2}}=c'\left(e\left(\gamma\right)\right). \tag{25}$$

For $\gamma=0$, equation (25) characterizes the equilibrium effort level for $\overline{N}+1$ agents, for $\gamma=1$, the solution to the equation is the equilibrium effort level when $\overline{N}+2$ agents are active.[9] Notice that (25) is well defined for any $\gamma\in[0,1]$. So we can apply the implicit function theorem and differentiate (25) with respect to γ

$$\frac{de}{d\gamma}=\frac{\frac{-2e(\gamma)}{\left(h+\left(\overline{N}+1+\gamma\right)e(\gamma)\right)^{3}}}{\frac{2\left(\overline{N}+1+\gamma\right)}{\left(h+\left(\overline{N}+1+\gamma\right)e(\gamma)\right)^{3}}+c''\left(e\left(\gamma\right)\right)}. \tag{26}$$

Consider first the case where $c(\cdot)$ is convex. In that case, the denominator of (26) is strictly positive so that adding another member clearly decreases individual incentives. Using the fact that $e^{*}\left(\overline{N}+1\right)-e^{*}\left(\overline{N}\right)=\int_{0}^{1}\frac{de}{d\gamma}d\gamma$, we have shown that individual equilibrium incentives are lower with more experts. But then, the equilibrium marginal cost of information acquisition on the right-hand side of (25) must decrease if there is one more member. This in turn requires that the equilibrium aggregate precision of information, $\left(\overline{N}+1+\gamma\right)e\left(\gamma\right)$, on the left-hand side of (25) must increase in γ. So adding one agent improves the quality of decision making (gross of costs of information acquisition). Consider next the case where $c(\cdot)$ is concave. By the assumptions introduced in section 3, the denominator of (26) still has a positive sign, so adding one member has detrimental effects on individual equilibrium incentives. However, now the right-hand side of (25) decreases when there is one more expert, which can only be the case if $\left(\overline{N}+1+\gamma\right)e\left(\gamma\right)$ is decreasing in γ. So, the quality of information worsens also in the aggregate.

In a similarly compact way of writing we can express (24) both in an equilibrium where $\overline{N}+1$ and $\overline{N}+2$

[9] $e(\gamma)$ is short notation for $e\left(\gamma,\overline{N},h\right)$. Since both h and $\overline{N}$ are fixed in what follows, we suppress the dependence on $\overline{N}$ and h.

agents gather information as

$$W\left(\overline{N}+\gamma\right)=-\frac{N}{h+\left(\overline{N}+1+\gamma\right)e\left(\gamma\right)}-\left(\overline{N}+1+\gamma\right)c\left(e\left(\gamma\right)\right) \tag{27}$$

The president should talk to one additional agent if and only if $W\left(\overline{N}+1\right)-W\left(\overline{N}\right)>0$. We have the following clear-cut result.

Proposition 3 *Group welfare is increasing in the number of experts if and only if there are decreasing returns to information acquisition. Formally,* $W\left(\overline{N}+1\right)-W\left(\overline{N}\right)\gtreqless 0$ *if and only if* $c''\left(\cdot\right)\gtreqless 0$.

Proof. Using condition (25), we can write

$$W\left(\overline{N}+\gamma\right)=-N\left(c'\left(e\left(\gamma\right)\right)\right)^{\frac{1}{2}}-\left(\overline{N}+1+\gamma\right)c\left(e\left(\gamma\right)\right).$$

So the difference in welfare when one additional agent gathers information can be stated as

$$\begin{aligned}&W\left(\overline{N}+1\right)-W\left(\overline{N}\right)\\=&\left\{\begin{array}{c}\left(-N\left(c'\left(e\left(\gamma\right)\right)\right)^{\frac{1}{2}}-\left(\overline{N}+1+\gamma\right)c\left(e\left(\gamma\right)\right)\right)\Big|_{\gamma=1}\\-\left(-N\left(c'\left(e\left(\gamma\right)\right)\right)^{\frac{1}{2}}-\left(\overline{N}+1+\gamma\right)c\left(e\left(\gamma\right)\right)\right)\Big|_{\gamma=0}\end{array}\right\}\\=&\int_0^1\left(-\frac{1}{2}N\left(c'\left(e\left(\gamma\right)\right)\right)^{-\frac{1}{2}}c''\left(e\left(\gamma\right)\right)\frac{de}{d\gamma}-\left(\overline{N}+1+\gamma\right)c'\left(e\left(\gamma\right)\right)\frac{de}{d\gamma}-c\left(e\left(\gamma\right)\right)\right)d\gamma\end{aligned}$$

where the second equality uses the fundamental theorem of integration. Substituting for $\frac{de}{d\gamma}$ from (26) and simplifying we can write

$$W\left(\overline{N}+1\right)-W\left(\overline{N}\right)=\int_0^1\left(\frac{Nc''\left(e\left(\gamma\right)\right)+2\left(\overline{N}+1+\gamma\right)c'\left(e\left(\gamma\right)\right)^{\frac{3}{2}}}{2\left(\overline{N}+1+\gamma\right)c'\left(e\left(\gamma\right)\right)^{\frac{3}{2}}+c''\left(e\left(\gamma\right)\right)}e\left(\gamma\right)c'\left(e\left(\gamma\right)\right)-c\left(e\left(\gamma\right)\right)\right)d\gamma$$

The fraction inside the brackets is larger (smaller) than one if and only if $c''\left(\cdot\right)>0$ $\left(c''\left(\cdot\right)<0\right)$. Moreover, $e\left(\gamma\right)c'\left(e\left(\gamma\right)\right)-c\left(e\left(\gamma\right)\right)$ is positive (negative) if and only if $c''\left(\cdot\right)>0$ $\left(c''\left(\cdot\right)<0\right)$. Taking the two facts together we have $W\left(\overline{N}+1\right)-W\left(\overline{N}\right)>0$ if and only if $c''\left(\cdot\right)>0$. So the integrand is bounded below pointwise by $e(\gamma)\, c'(e(\gamma)) - c(e(\gamma))$, which is positive by convexity of $c(\cdot)$. ■

Thus, for the case of convex costs, the free-rider effect is outweighed by the increased aggregate quality of information and the improved distribution of costs of information acquisition. In case of concave costs of information acquisition, the aggregate quality of information and the distribution of costs gets worse.

Since the group has identical ex post interests there are strong incentives to coordinate on the most efficient outcome. For this reason it is interesting to know how the best equilibrium behaves as a function of the group size. Two questions seem relevant. First, do agents who are already in the group benefit if an additional agent joins the group? Second, does the incentive problem get worse or is it ameliorated if there are more agents? We have the following results:

Proposition 4 *Suppose that $c(\cdot)$ is convex. Then, per capita utility is increasing in the number of agents N. As N tends to infinity, per capita utility tends to its first-best value. If $c(\cdot)$ is concave, then per capita utility is increasing in N but bounded away from its first-best value.*

The proof of these statements is similar to the one of proposition three and therefore relegated to the appendix. For the case of decreasing returns to scale, per capita utility is increasing in the number of agents in the group. The reason is again that the aggregate precision of information is increased when one more agent gathers information. In addition the cost of information acquisition per head is decreased since all agents reduce the precision of their individual pieces of information. However, it is interesting to observe that per capita utility increases less than one for one with group size. So group utility is actually decreasing when an additional agent joins (and this is true both in the first- and in the second best). Finally, when the group gets very large, the inefficiency of the equilibrium disappears, i.e., per capita utility goes to its first-best value. In the limit as N goes out of bounds, each agent's first-best precision of information is close to zero. So the first-best and the equilibrium choices of precisions converge to each other.

In case costs are concave, welfare is maximized when only one agent acquires information. But then the only effect of increasing the number of agents in the group is to exchange (ex ante) payments to spread the monetary equivalent of the cost of effort among the members of the group. So, taking account of payments, every member of the group bears approximately zero costs of information acquisition when N gets large. However, the amount of information acquired by the expert is independent of the number of agents in the group and inefficiently low, so the first-best utility cannot be reached.

In the remainder of this article, we extend these basic results in various directions. Taking a team-theory perspective (see Marshak and Radner [18]), we focus on the most efficient equilibrium in each application that we consider. Since the case of decreasing returns to information acquisition is much richer in this respect, we assume from now on that $c(\cdot)$ is convex.

6 Multiple Tasks

Suppose there are many decisions to take. More specifically, let there be $l = 1, ..., L$ decisions and the information gathered about one decision be independent of information gathered about any other decision. All prior means are identical, and the prior precisions are $h^l = h$ for $l = 1, ..., L$. Suppose $L = N$. Finally, suppose that the agents' costs of information acquisition just depends on the sum of an agent's efforts for all tasks. Is it better to assign each agent to one of the tasks or to share the responsibility for all tasks equally among all agents?

If each agent is responsible for exactly one decision, then -in the efficient equilibrium - the agent, say agent i, who is responsible for decision l acquires information to the point where

$$\frac{1}{\left(h + e_{il}^*\right)^2} = c'\left(e_{il}^*\right)$$

Clearly, all agents exert the same effort level, so $e_{il}^* = e^*$ for all i and l. Let γ denote the number of decisions each individual is involved in. Let $W(\gamma)$ denote welfare as a function of the number of tasks assigned to each agent. In case of single-tasking, $\gamma = 1$, welfare is equal to

$$W(1) = -N\frac{N}{h + e^*} - Nc\left(e^*\right)$$

since there are $L = N$ decisions to take.

In contrast, when all agents share the responsibility for all the tasks each agent acquires information for decision l to the point where

$$\frac{1}{\left(h+\sum_{-i} e_{jl}+e_{il}\right)^2} = c'\left(\sum_l e_{il}\right)$$

Such a condition must hold for all $i = 1, ..., N$ and all $l = 1, ..., L$. Since the marginal cost depends only on the sum of all efforts spent on all problems, it follows that the marginal values of additional information must be equalized across decisions, so

$$\frac{1}{\left(h+\sum e_{jl'}\right)^2} = \frac{1}{\left(h+\sum e_{jl''}\right)^2}$$

for all l' and l''. By the now familiar argument, the effort levels of all individuals working on the same task must be identical. Again, the reason for this result is that the marginal value of information concerning the decision l depends only on the sum of precisions of information gathered about problem l. As a result, there is a unique equilibrium constellation, where all individuals gather information of identical precision for all the l decisions. So, the equilibrium precision of information satisfies the condition

$$\frac{1}{\left(h+Ne^*\right)^2} = c'\left(Ne^*\right)$$

and welfare is equal to

$$W\left(N\right) = -N\frac{N}{h+Ne^*} - Nc\left(Ne^*\right)$$

We can now state the following result.

Proposition 5 *Sharing responsibility on each task among all agents or having one agent responsible per task are welfare equivalent.*

Proof. To compare welfare across the two systems, observe again that the equilibrium precision of information in both games has the common representation

$$\frac{1}{\left(h+\gamma e^*\left(\gamma\right)\right)^2} = c'\left(\gamma e^*\left(\gamma\right)\right)$$

where $\gamma = 1$ for the equilibrium where each agent is responsible for a single task and $\gamma = N$ where responsibility is shared equally among all agents. Similarly, welfare can be written as

$$W(\gamma) = -N \frac{N}{h + \gamma e^*(\gamma)} - Nc(\gamma e^*(\gamma))$$

Differentiating and integrating up we find

$$\begin{aligned} W(N) - W(1) &= \int_1^N \left[\begin{array}{c} \left(\frac{N^2 \gamma}{(h+\gamma e^*(\gamma))^2} - N\gamma c'(\gamma e^*(\gamma)) \right) \frac{de^*}{d\gamma} + \frac{N^2 e^*(\gamma)}{(h+\gamma e^*(\gamma))^2} \\ -Nc'(\gamma e^*(\gamma)) e^*(\gamma) \end{array} \right] d\gamma \\ &= \int_1^N \left[\left(\frac{N^2}{(h+\gamma e^*(\gamma))^2} - Nc'(\gamma e^*(\gamma)) \right) \left(\gamma \frac{de^*}{d\gamma} + e^*(\gamma) \right) \right] d\gamma \end{aligned}$$

But, from the equilibrium condition for the precision levels we find that

$$\frac{de^*}{d\gamma} = \frac{\left(\frac{2}{(h+\gamma e^*(\gamma))^3} + c''(\gamma e^*(\gamma)) \right) e^*(\gamma)}{\left(\frac{-2}{(h+\gamma e^*(\gamma))^3} - c''(\gamma e^*(\gamma)) \right) \gamma} = -\frac{e^*(\gamma)}{\gamma}$$

so that $\gamma \frac{de^*}{d\gamma} + e^* = 0$. It follows that $W(N) - W(1) = 0$. ■

If there is one and only one agent who is responsible for one task, then this agent acquires more precise information for his task. But it turns out that this higher precision per individual is exactly compensated by the reduction in the number of signals used per problem. Formally, the aggregate precision of new information per task is equal under both forms of organizing expertise. As a result, both the cost of acquiring the information and the expected utility from decision-making are identical in both situations. So, there is no difference in terms of welfare between the two forms either.

Note that this result depends on the way the cost of information acquisition is modeled. Suppose we had assumed that the total cost of an agent who works on all tasks is $\sum_{l=1}^{L} c(e_{il})$. In some sense the agent has different mental accounts for each task. Obviously, the problem is separable in this case. We have replicated the original problem L-times, but there is no interaction at all among these L different games played by the agents. So, our original results carry over, so that a single expert on each task is optimal with increasing returns, and all agents sharing responsibility for all the tasks is optimal with decreasing returns.

7 Communication with Bounded Receiver Capacity

We now introduce constraints on the capability of the president to communicate with agents, or more generally, to digest information she receives. Suppose the president can only use $K < N$ signals, including her own. (Nothing is lost if we would assume that the president can use her own information without making use of scarce capacity. However, notation would be somewhat more cumbersome.) Obviously, this analysis is interesting only in case there are decreasing returns to information acquisition at the individual level, which we assume in this section.

Applying a standard decreasing returns assumption it is easy to see with whom the president will communicate. Suppose the president conjectures that the agents hold information of differing precisions. If there are K agents, all of which have information that is more precise than possessed by the $N-K$ remaining agents, the president should talk to the former group only. If not, suppose there are $k < K$ agents whose information is more precise than the one possessed by the $N-k$ other agents individually. Then, the president should talk for sure to the first k agents. By definition, she will be indifferent whom to talk to within this group of $N-k$ agents, so we let her randomize with equal probability among these agents. Since there are $N-k$ agents left, the probability that she will talk to any individual agent in the latter group is $\frac{K-k}{N-k}$. Vice versa, all k agents with whom the president communicates with probability one face the same incentives, so their equilibrium precision of information must be identical and equal to some $\overline{e}(k, K)$. We will call agents in this group the important agents. Likewise, the remaining $N-k$ agents with whom the president communicates with probability $\frac{K-k}{N-k}$ all face identical incentives, so their equilibrium precision of information is identical and equal to, say $\underline{e}(k, K)$. Agents in this group will be called unimportant.

Clearly, like the game without capacity constraints, this game has babbling equilibria. We focus on the most informative equilibrium, as is standard in team theory. Consider first agents with whom the president communicates with probability one. Their information always has an impact on the decision made. In addition, they know that $k-1$ other agents face the same incentives and $K-k$ agents contribute information of a differing precision, which is however, identical for all these agents. So, their optimal choice of precision satisfies the first-order condition

$$\frac{1}{\left(h+k\overline{e}\left(k,K\right)+\left(K-k\right)\underline{e}\left(k,K\right)\right)^{2}}=c'\left(\overline{e}\left(k,K\right)\right). \tag{28}$$

Agents in the second group recognize that the decision taken will always make use of k signals of precision $\overline{e}(k,K)$ and $K-k$ signals of precision $\underline{e}(k,K)$. However, their own information influences the decision made only with probability $\frac{K-k}{N-k}$. Thus, their optimal choice of precision satisfies the first-order condition

$$\frac{K-k}{N-k}\frac{1}{\left(h+k\overline{e}\left(k,K\right)+\left(K-k\right)\underline{e}\left(k,K\right)\right)^{2}}=c'\left(\underline{e}\left(k,K\right)\right). \tag{29}$$

From these two conditions it is immediate that the two precision levels satisfy $\frac{N-k}{K-k}c'\left(\underline{e}\left(k,K\right)\right)=c'\left(\overline{e}\left(k,K\right)\right)$, or in other words

$$\underline{e}\left(k,K\right)=c'^{-1}\left(\frac{K-k}{N-k}c'\left(\overline{e}\left(k,K\right)\right)\right). \tag{30}$$

Equation (30) is an equilibrium relation between the effort levels of important and unimportant agents, respectively. Since the right hand side of the equation is increasing in $\overline{e}(k,K)$, the relation is a function. Substituting back into the first-order condition of important agents we have the equilibrium condition

$$\frac{1}{\left(h+k\overline{e}\left(k,K\right)+\left(K-k\right)c'^{-1}\left(\frac{K-k}{N-k}c'\left(\overline{e}\left(k,K\right)\right)\right)\right)^{2}}=c'\left(\overline{e}\left(k,K\right)\right). \tag{31}$$

It is easy to see that there exists a unique solution to equation (31). The expression on the left-hand side is decreasing in $\overline{e}(k,K)$ and takes a value of h^{-2} at $\overline{e}(k,K)=0$. On the other hand, the expression on the right-hand side is increasing in $\overline{e}(k,K)$ and takes a value of zero at $\overline{e}(k,K)=0$.

7.1 Incentive Effects of Asymmetric Expertise

To develop an intuition of how k influences these effort levels, we begin by investigating how a change in k affects the effort level of important and unimportant agents, respectively, when the effort level of the other group is kept constant. An analytical convenient way to do this is again achieved by a convexification. Define

$\hat{k} = k + \gamma$. For $\gamma = 0$ $\hat{k} = k$, the number of important agents in the base situation. For $\gamma = 1$, $\hat{k} = k + 1$, the number of agents when there is none more important agent. Convexify the domain of definition of γ to the unit interval. We can then apply the implicit function theorem to see how an increase in $\hat{k}$ impacts on the precision levels of important and unimportant agents, respectively. From (28), holding $\underline{e}\left(\hat{k}, K\right)$ constant, we get

$$\frac{d\overline{e}}{d\hat{k}} = \frac{\frac{-2\left(\overline{e}\left(\hat{k},K\right)-\underline{e}\right)}{\left(h+\hat{k}\overline{e}\left(\hat{k},K\right)+\left(K-\hat{k}\right)\underline{e}\right)^3}}{\frac{2\hat{k}}{\left(h+\hat{k}\overline{e}\left(\hat{k},K\right)+\left(K-\hat{k}\right)\underline{e}\right)^3} + c''\left(\overline{e}\left(\hat{k}, K\right)\right)} < 0$$

for $\overline{e}(k, K) - \underline{e} > 0$, which will be the case around the equilibrium, and

$$\frac{d\overline{e}}{d\underline{e}} = \frac{\frac{-2(K-k)}{(h+k\overline{e}(k,K)+(K-k)\underline{e})^3}}{\frac{2k}{(h+k\overline{e}(k,K)+(K-k)\underline{e})^3} + c''\left(\overline{e}(k, K)\right)} < 0.$$

If there are more important agents, then each one of them will have less of an incentive to work. Similarly, if other agents (the unimportant ones) provide more effort, then important agents have less of an incentive to work.

Similarly, the unimportant agents' incentives change in the following way with a change in $\hat{k}$. From (29), holding $\overline{e}(k, K)$ constant, we have

$$\frac{d\underline{e}}{d\hat{k}} = \frac{\left(-\frac{N-K}{\left(N-\hat{k}\right)^2}\frac{1}{\left(h+\hat{k}\overline{e}+\left(K-\hat{k}\right)\underline{e}\left(\hat{k},K\right)\right)^2} - \frac{K-\hat{k}}{N-\hat{k}}\frac{2\left(\overline{e}-\underline{e}\left(\hat{k},K\right)\right)}{\left(h+\hat{k}\overline{e}+\left(K-\hat{k}\right)\underline{e}\left(\hat{k},K\right)\right)^3}\right)}{\frac{K-\hat{k}}{N-\hat{k}}\frac{2\left(K-\hat{k}\right)}{\left(h+\hat{k}\overline{e}+\left(K-\hat{k}\right)\underline{e}\left(\hat{k},K\right)\right)^3} + c''\left(\underline{e}\left(\hat{k}, K\right)\right)} < 0$$

again for the relevant case where $\overline{e} - \underline{e}\left(\hat{k}, K\right) > 0$, and

$$\frac{d\underline{e}}{d\overline{e}} = \frac{-\frac{K-k}{N-k}\frac{2(K-k)}{(h+k\overline{e}+(K-k)\underline{e}(k,K))^3}}{\frac{K-k}{N-k}\frac{2(K-k)}{(h+k\overline{e}+(K-k)\underline{e}(k,K))^3} + c''\left(\underline{e}(k, K)\right)} < 0.$$

So the unimportant agents' incentives are reduced if there are more important agents. Similarly, if the group of important agents provides information of higher quality, then the unimportant agents have less incentives to generate high quality information.

7.2 Welfare Analysis

We can now compare the welfare properties across the different equilibria induced by a different choice of k. First, we compare the extremes $k = 0$ and $k = K$. Since these equilibria are essentially symmetric (in the sense of symmetry among agents that are actually active), the first-order conditions in these two situations have the common representation

$$\gamma \frac{1}{(h + Ke(\gamma))^2} = c'(\overline{e}(\gamma)) \tag{32}$$

where $\gamma = 1$ is the case where $k = K$ and $\gamma = \frac{K}{N}$ represents the case where $k = 0$. Similarly, welfare can be written as

$$W(K, \gamma) = -\frac{N}{h + Ke(\gamma)} - \frac{K}{\gamma} c(e(\gamma)) \tag{33}$$

We can state the following result.

Proposition 6 *Welfare in the asymmetric equilibrium with K experts and $N - K$ ignorants is higher than in the symmetric equilibrium solution where the president communicates with all agents with probability $\frac{K}{N}$.*

Proof. Extend the domain of definition of γ to the interval $\left[\frac{K}{N}, 1\right]$. Differentiating (33) with respect to γ, we obtain

$$\frac{\partial W(K, \gamma)}{\partial \gamma} = \left(\frac{NK}{(h + Ke(\gamma))^2} - \frac{K}{\gamma} c'(e(\gamma)) \right) \frac{de}{d\gamma} + \frac{K}{\gamma^2} c(e(\gamma))$$

Applying the implicit function theorem to equation (32) we find that

$$\frac{de}{d\gamma} = \frac{\frac{1}{(h+Ke(\gamma))^2}}{\left(\gamma \frac{2K}{(h+Ke(\gamma))^3} + c''(\overline{e}(\gamma))\right)} > 0$$

it follows that

$$W(1) - W\left(\frac{K}{N}\right) = \int_{\frac{K}{N}}^{1} \frac{\partial W(K, \gamma)}{\partial \gamma} d\gamma = \int_{\frac{K}{N}}^{1} \left(K(N-1) c'(e(\gamma)) \frac{1}{\gamma} \frac{de}{d\gamma} + \frac{K}{\gamma^2} c(e(\gamma)) \right) > 0.$$

■

When the president treats everybody symmetrically, then all agents have weak incentives and the aggregate quality of information is low. On the other hand, the cost of information acquisition is different in the two situations. However, since there are externalities and all agents neglect the public nature of information, an increase in the aggregate quality of information is beneficial. Therefore, it is better to have K experts rather than a symmetric treatment of individuals.

7.3 Hybrid Equilibria

Although the logic in the previous argument does not in any essential way rely on the symmetry of the equilibria across active agents, symmetry is quite helpful analytically. In particular, a welfare comparison across all equilibria is very messy in general. However, for a particular class of cost functions we can completely order all the equilibria. Consider the two first-order conditions of important and unimportant agents, respectively. For the case where $c(e) = \frac{e^{\eta}}{\eta}$, the equation $\frac{N-k}{K-k} c'(\underline{e}(k, K)) = c'(\overline{e}(k, K))$ takes the form $\frac{N-k}{K-k} \underline{e}^{\eta-1} = \overline{e}^{\eta-1}$. So, the equilibrium precision of unimportant agents' information becomes linear in the precision of important agents' information:

$$\underline{e} = \left(\frac{K-k}{N-k} \right)^{\frac{1}{\eta-1}} \overline{e}.$$

This allows us to eliminate one precision level from the equilibrium condition. Then, a complete welfare analysis of the entire equilibrium set is feasible, with the following conclusion:

Proposition 7 *Welfare in the equilibrium with maximum asymmetry between important and unimportant agents, i.e., when $k = K$, is higher than the welfare in any equilibrium induced by any $k < K$.*

The proof of this statement is relegated to the appendix. The intuition is straightforward. Although going into the direction of a more symmetric situation improves the distribution of costs in the group of agents, it generates a certain waste. Since only K signals can be used, if there is one more less important person, then one less signal of relatively high precision can be used in decision-making and is replaced with a less precise one. So the aggregate precision of the information that is actually used in decision-making is reduced. And this negative effect outweighs the positive cost-saving effect.

8 Noisy Communication among Heterogeneous Agents

Suppose the agents differ in their ability to gather information. In particular, suppose we can order the agents according to their marginal costs of acquiring information in the sense that $c_i'(e) < c_j'(e)$ for all e and $i > j$. In words, agent N is most efficient at acquiring information, agent 1 is least efficient. For reasons of space we assume that information is verifiable in this section[10]. Then, the following statement is a direct corollary of proposition 1.

Proposition 8 *There exist* 2^{N-1} *equilibria, where agents are partitioned into a passive and an expert group. Within the expert group, agents with a higher index acquire more precise information and receive a higher weight in equilibrium than agents with a lower index.*

The proof of this statement is obvious and therefore omitted. The point is that the equilibrium precisions of agent's i and j satisfy

$$\frac{1}{(h + A(\mathbf{e}^*))^2} = c_i'(e_i^*) = c_j'\left(e_j^*\right)$$

Since the marginal value of information must be the same for all active agents, so must the marginal costs be. Given that these marginal costs can be ordered, we get a hierarchy of importance among agents. It should be noted that exactly the same result could be obtained letting the agents have identical cost functions but let them have differing intensities attached to their loss functions. What is important is that the marginal rates of substitution of the agents between losses from risky decision-making and costs of information acquisition can be ordered.

Introduce now noise into the communication. Suppose if an agents sends the message s_j the president understands $\overline{s}_j = s_j + \varepsilon_p$ where $\tilde{\varepsilon}_p \sim N\left(0, \frac{1}{p_j}\right)$, independently of ε_j (all j) and y. For simplicity we take p_j proportional to e_j and let $p_j = \lambda e_j$. These assumptions imply that $\overline{s}_j \,|\, y \sim N\left(0, \frac{1}{\frac{\lambda}{1+\lambda} e_j}\right)$. So, communicating information of precision e_j through the noisy channel is equivalent to communicating information of precision $\frac{\lambda}{1+\lambda} e_j$ without noise. Let $\pi = \frac{\lambda}{1+\lambda}$ to simplify notation. If agent i is the president, and we focus on the most

[10] We conjecture that none of the results depend on verifiability. However, with nonverifiable information, we would have to reprove proposition 2 for asymmetric constellations, from which we abstain.

efficient equilibrium where all agents are active, the equilibrium precision of information satisfies

$$\left(h + e_i^* + \pi \sum_{j \neq i} e_j^*\right)^{-2} = c_i'\left(e_i^*\right) = \frac{c_j'\left(e_j^*\right)}{\pi} \tag{34}$$

The difference between the president (apart from differences in the cost of acquiring information) is that her information is used as precise as it is. In contrast, the other agents take into account that their information is discounted, because noise is added to it through communication. We can raise a simple, but important question: who should be the president? To obtain a clear answer to this question, we assume that $c_j\left(e_j\right) = \frac{z_j}{\eta} e_j^\eta$ for all j and consider the limiting case where $h \to 0$. We find the following:

Proposition 9 *It is optimal to make the agent who is most efficient at acquiring information the president.*

Controlling for differences in costs of acquiring information, the president has the strongest incentive to acquire information, since none of his own information is lost. Welfare turns out to be proportional to the precision of information in the hands of the president. And this measure is maximized when the most efficient agent is the president. The proof of this statement is in the appendix.

9 Conclusion

We study a game of cheap talk with many agents, who gather costly information before communicating with each other. We characterize all fully informative equilibria where all the information that is acquired is also transmitted. Although information is of a continuous quality, in equilibrium, agents are partitioned into exactly two classes, one being irrelevant and one important. The important agents all have the same influence on the decision. We provide a complete welfare ranking of all equilibria as a function of the technology of information acquisition.

Throughout our analysis we have assumed that there is no commitment to the rules of choosing actions. This assumption can be relaxed without affecting the qualitative nature of our results within a class of decision rules. Arean [2] studies the optimal design of linear aggregation rules with verifiable information and convex costs of information acquisition, showing that the optimal solution is symmetric among agents.

In this case, each agent's information receives a weight that is greater than the statistically optimal weight at the expense of a smaller weight given to prior information.[11]

Our applications to task assignment, and to costly and noisy communication are only a few of the potential ones of our model. We leave others to future work.

10 Appendix

Proof of Lemma 1. Consider agent i's best reply to all other agents choosing $e_j = 0$. If $c'(0) < h^{-2}$, then agent i's payoff is increasing in e_i around $e_i = 0$. Since his payoff is continuous in e_i, and his payoff cannot increase without bounds, there must exist a stationary point to his payoff function, satisfying

$$\frac{1}{(h+\underline{e})^2} = c'(\underline{e})$$

Consider now any other agent $j \neq i$. Taking $\mathbf{e}_{-j}$ as given, agent j chooses $e_j(\mathbf{e}_{-j}) = 0$ if $c'(0) \geq \frac{1}{(h+\underline{e})^2}$. Since his payoff function is decreasing around $e_j = 0$, his payoff is decreasing for all e_j. To see this, suppose the contra-positive were true. Then, his payoff must eventually increase. But to achieve this, there must be a stationary point that corresponds to a minimum. But this contradicts the fact that any stationary point must be a maximum.

If $c'(0) < \frac{1}{(h+\underline{e})^2}$ then $e_j(\mathbf{e}_{-j})$ must satisfy

$$\frac{1}{(h + e_j(\mathbf{e}_{-j}) + \underline{e})^2} = c'(e_j(\mathbf{e}_{-j})) \tag{35}$$

building on analogous arguments as above.

If both agent i and j reply optimally to their effort choices, respectively, then they must both choose the same effort level, because the left-hand side of (35) depends only on the sum of efforts. Thus, in a potential

[11] See also Li [16] for a similar result.

equilibrium with two agents acquiring a positive amount of information, we should have

$$\frac{1}{(h+2\overline{e})^2} = c'(\overline{e})$$

We will now show that this cannot be an equilibrium, because the remaining agents do not play best replies.

More generally, let $\overline{N} \geq 1$ and $\overline{N}+1$ denote numbers of agents that choose a symmetric candidate equilibrium level of effort satisfying the condition

$$\frac{1}{\left(h+\left(\overline{N}+\gamma\right)e(\gamma)\right)^2} = c'(e(\gamma)) \tag{36}$$

for $\gamma = 0$ and $\gamma = 1$, respectively. We now show that the left-hand side is increasing in the number of agents that acquire information. To see this, convexify the domain of definition of γ to the unit interval. Then, we can treat $e(\gamma)$ as differentiable in γ and apply the implicit function theorem to obtain

$$\frac{de}{d\gamma} = \frac{\frac{2e(\gamma)}{\left(h+\left(\overline{N}+\gamma\right)e(\gamma)\right)^3}}{-\frac{2\left(\overline{N}+\gamma\right)}{\left(h+\left(\overline{N}+\gamma\right)e(\gamma)\right)^3} - c''(e(\gamma))}$$

The denominator can we written as $-2\left(\overline{N}+\gamma\right)\left(c'(e(\gamma))\right)^{\frac{3}{2}} - c''(e(\gamma))$, which is negative by assumption for all $\gamma \in [0,1]$. By the fundamental theorem of calculus, we have $e\left(\overline{N}+1\right) - e\left(\overline{N}\right) = \int_0^1 \frac{de}{d\gamma} d\gamma$. But then we have shown that $e\left(\overline{N}+1\right) < e\left(\overline{N}\right)$. By the fact that $c''(\cdot) < 0$, this implies that $c'\left(e\left(\overline{N}+1\right)\right) > c'\left(e\left(\overline{N}\right)\right)$. Using (36), this implies $\frac{1}{\left(h+\left(\overline{N}+1\right)e\left(\overline{N}+1\right)\right)^2} > \frac{1}{\left(h+\overline{N}e\left(\overline{N}\right)\right)^2}$. But then, $c'(0) < \frac{1}{(h+\underline{e})^2}$ implies that $c'(0) < \frac{1}{\left(h+\overline{N}e\left(\overline{N}\right)\right)^2}$ for any $\overline{N}$, so that in equilibrium all the agents must acquire the same, positive amount of information. ■

Proof of Proposition 2. Pick a putative equilibrium where $\overline{N} \leq N-1$ agents plus the president are supposed to exert effort level $e^*\left(\overline{N}\right)$. Since all agents are identical ex ante and supposed to behave identically in the game, their labels are arbitrary. So reorder the agents as follows. Let $i = 2, ..., \overline{N}$ be the honest agents who spend effort $e^*\left(\overline{N}\right)$ and report truthfully. Let $i = \overline{N}+1$ denote the president, and let $i = 1$ denote the agent who potentially shirks. Finally, in case $\overline{N} < N-1$, $i = \overline{N}+2, ..., N$ denotes agents that are supposed to exert effort $e = 0$ in the putative equilibrium. Since these agents are irrelevant in decision-making, we neglect them in this proof. We let $\mathbf{e}^*_{-1}$ denote the vector that has $\overline{N}$ entries $e^*\left(\overline{N}\right)$. With truthful strategies for the first $\overline{N}$ agents, we can identify $m_i \equiv s_i$ for $i = 2, ..., \overline{N}+1$. Let $\mathbf{s}_{-1}$ denote the $\overline{N}$-vector of signals excluding signal s_1. Using (8), the decision function can be written

$$x\left(m_1, \mathbf{s}_{-1}, \alpha^*\right) = (1 - \sum_{i=1}^{\overline{N}+1} \alpha^*)\mu + \alpha^* m_1 + \sum_{i=2}^{\overline{N}+1} \left(\alpha^* s_i\right) \tag{37}$$

where $\alpha^* \equiv \alpha^*\left(\mathbf{e}\right) = \frac{e^*\left(\overline{N}\right)}{h+\left(\overline{N}+1\right)e^*\left(\overline{N}\right)}$. Notice that the right-hand side of (37) coincides with the conditional expectation of y given the signals and effort if and only if agent one exerts effort $e_1 = e^*\left(\overline{N}\right)$ and sends message $m_1 = s_1$ for all s_1. If agent 1 deviates and chooses an effort level $\hat{e}_1 \neq e^*\left(\overline{N}\right)$ then the true conditional expectation is

$$E\left[y|\,\mathbf{s}; \mathbf{e}^*_{-1}, \hat{e}_1\right] = (1 - \hat{\alpha}_1 - \sum_{i=2}^{\overline{N}+1} \hat{\alpha}_{-1})\mu + \hat{\alpha}_1 s_1 + \sum_{i=2}^{\overline{N}+1} \left(\hat{\alpha}_{-1} s_i\right) \tag{38}$$

where $\hat{\alpha}_1 = \frac{\hat{e}_1}{h+\hat{e}_1+\overline{N}e^*\left(\overline{N}\right)}$ and $\hat{\alpha}_{-1} = \frac{e^*\left(\overline{N}\right)}{h+\hat{e}_1+\overline{N}e^*\left(\overline{N}\right)}$. A deviation of agent 1 changes the statistically optimal weight of all signals.

Let $f\left(y, \mathbf{s}_{-1}\left|s_1, \mathbf{e}^*_{-1}, \hat{e}_1\right.\right)$ denote agent 1′s posterior about y and $\mathbf{s}_{-1}$ and let $g\left(\mathbf{s}_{-1}\left|s_1, \mathbf{e}^*_{-1}, \hat{e}_1\right.\right)$ denote the conditional density of $\mathbf{s}_{-1}$ given agent 1′s information. Notice that $f\left(y, \mathbf{s}_{-1}\left|s_1, \mathbf{e}^*_{-1}, \hat{e}_1\right.\right) = f\left(y\left|\mathbf{s}, \mathbf{e}^*_{-1}, \hat{e}_1\right.\right) g\left(\mathbf{s}_{-1}\left|s_1, \mathbf{e}^*_{-1}, \hat{e}_1\right.\right)$. With a slight abuse of notation let $x\left(\alpha^*, m_1, \mathbf{s}_{-1}\right)$ denote the president's aggregation rule. Let

$$\begin{aligned} & V\left(\alpha^*, m_1, \mathbf{e}^*_{-1}, \hat{e}_1\right) \\ \equiv\; & -\int_{-\infty}^{\infty}\int\cdots\int_{-\infty}^{\infty}\left(x\left(\alpha^*, m_1, \mathbf{s}_{-1}\right) - y\right)^2 f\left(y\left|\mathbf{s}, \mathbf{e}^*_{-1}, \hat{e}_1\right.\right) g\left(\mathbf{s}_{-1}\left|s_1, \mathbf{e}^*_{-1}, \hat{e}_1\right.\right) dy ds_2 \cdots ds_{\overline{N}+1} \end{aligned}$$

denote agent 1's conditional expected utility given his information, and given his effort choice is $\hat{e}_1$ and the effort choice of all other relevant agents is $e^*\left(\overline{N}\right)$. Completing the square by $E\left[y|\,\mathbf{s}; \mathbf{e}^*_{-1}, \hat{e}_1\right] - E\left[y|\,\mathbf{s}; \mathbf{e}^*_{-1}, \hat{e}_1\right]$

and observing that $\int_{-\infty}^{\infty}\left(E\left[y|\,\mathbf{s};\mathbf{e}_{-1}^{*},\hat{e}_{1}\right]-y\right)f\left(y\left|\mathbf{s},\mathbf{e}_{-1}^{*},\hat{e}_{1}\right.\right)dy=0$ we can write

$$\begin{aligned} & V\left(\alpha^{*},m_{1},\mathbf{e}_{-1}^{*},\hat{e}_{1}\right) \\ = & -\int_{-\infty}^{\infty}\int\cdots\int_{-\infty}^{\infty}\left(\begin{array}{c}\left(x\left(\alpha^{*},m_{1},\mathbf{s}_{-1},\right)-E\left[y|\,\mathbf{s};\mathbf{e}_{-1}^{*},\hat{e}_{1}\right]\right)^{2} \\ +\left(E\left[y|\,\mathbf{s};\mathbf{e}_{-1}^{*},\hat{e}_{1}\right]-y\right)^{2}\end{array}\right)f\left(y\left|\mathbf{s},\mathbf{e}_{-1}^{*},\hat{e}_{1}\right.\right)g\left(\mathbf{s}_{-1}\left|s_{1},\mathbf{e}_{-1}^{*},\hat{e}_{1}\right.\right)dyds_{2}\cdots ds_{\overline{N}+1}\end{aligned}$$

Notice that $V\left(\alpha^{*},m_{1},\mathbf{e}_{-1}^{*},\hat{e}_{1}\right)$ is a strictly concave function of m_1. Thus, the first-order condition is necessary and sufficient for the unique optimal report. Thus, the report is optimal iff $\frac{\partial V\left(\alpha^{*},m_{1}^{*},\mathbf{e}_{-1}^{*},\hat{e}_{1}\right)}{\partial m_{1}}=0$, i.e.

$$-\int\cdots\int_{-\infty}^{\infty}2\alpha^{*}\left(x\left(\alpha^{*},m_{1},\mathbf{s}_{-1}\right)-E\left[y|\,\mathbf{s};\mathbf{e}_{-1}^{*},\hat{e}_{1}\right]\right)g\left(\mathbf{s}_{-1}\left|s_{1},\mathbf{e}_{-1}^{*},\hat{e}_{1}\right.\right)ds_{2}\cdots ds_{\overline{N}+1}=0$$

Using (37) and (38), and integrating, the first-order condition becomes

$$\left(\alpha^{*}\left(m_{1}^{*}-\mu\right)-\hat{\alpha}_{1}\left(s_{1}-\mu\right)+\sum_{i=2}^{\overline{N}+1}\left(\left(\alpha^{*}-\hat{\alpha}_{-1}\right)\left(E\left[s_{i}\left|s_{1};\mathbf{e}_{-1}^{*},\hat{e}_{1}\right.\right]-\mu\right)\right)\right)=0$$

Using standard results, we note that $E\left[s_{i}\left|s_{1};\mathbf{e}_{-1}^{*},\hat{e}_{1}\right.\right]=E\left[s_{i}\right]+\frac{Cov\left[s_{i},s_{1}\right]}{Var\left(s_{1}\right)}\left(s_{1}-E\left[s_{1}\right]\right)=\mu+\frac{\frac{1}{h}}{\frac{1}{h}+\frac{1}{\hat{e}_{1}}}\left(s_{1}-\mu\right)=\mu+\frac{\hat{e}_{1}}{h+\hat{e}_{1}}\left(s_{1}-\mu\right).$

Compute now the indirect utility arising from the optimal lie.

$$\begin{aligned} & E_{\mathbf{s}_{-1}}\left[\left.\left(x\left(\alpha^{*},m_{1},\mathbf{s}_{-1}\right)-E\left[y|\,\mathbf{s};\mathbf{e}_{-1}^{*},\hat{e}_{1}\right]\right)^{2}\right|s_{1},\mathbf{e}\right] \\ = & E_{\mathbf{s}_{-1}}\left[\left.\left((1-\sum_{i=1}^{\overline{N}+1}\alpha^{*})\mu+\alpha^{*}m_{1}^{*}+\sum_{i=2}^{\overline{N}+1}(\alpha^{*}s_{i})-\left((1-\hat{\alpha}_{1}-\sum_{i=2}^{\overline{N}+1}\hat{\alpha}_{-1})\mu+\hat{\alpha}_{1}s_{1}+\sum_{i=2}^{\overline{N}+1}(\hat{\alpha}_{-1}s_{i})\right)\right)^{2}\right|s_{1},\mathbf{e}\right] \\ = & E_{\mathbf{s}_{-1}}\left[\left.\left(\alpha^{*}\left(m_{1}^{*}-\mu\right)+\sum_{i=2}^{\overline{N}+1}\left(\alpha^{*}-\hat{\alpha}_{-1}\right)\left(s_{i}-\mu\right)-\hat{\alpha}_{1}\left(s_{1}-\mu\right)\right)^{2}\right|s_{1},\mathbf{e}_{-1}^{*},\hat{e}_{1}\right] \\ = & E_{\mathbf{s}_{-1}}\left[\left.\left(\sum_{i=2}^{\overline{N}+1}\left(\alpha^{*}-\hat{\alpha}_{-1}\right)\left(s_{i}-E\left[s_{i}\left|s_{1};\mathbf{e}_{-1}^{*},\hat{e}_{1}\right.\right]\right)\right)^{2}\right|s_{1},\mathbf{e}_{-1}^{*},\hat{e}_{1}\right] \\ = & \sum_{i=2}^{\overline{N}+1}\left(\alpha^{*}-\hat{\alpha}_{-1}\right)^{2}Var\left[s_{i}\left|s_{1};\mathbf{e}_{-1}^{*},\hat{e}_{1}\right.\right]+2\sum_{i=2}^{\overline{N}}\sum_{j=i+1}^{\overline{N}+1}\left(\alpha^{*}-\hat{\alpha}_{-1}\right)^{2}Cov\left[s_{i},s_{j}\left|s_{1};\mathbf{e}_{-1}^{*},\hat{e}_{1}\right.\right]\end{aligned}$$

It is easy to show that $Var\left[s_i\left|s_1;\mathbf{e}_{-1},\hat{e}_1\right.\right]=\frac{1}{h}+\frac{1}{e_i}-\frac{\frac{1}{h^2}}{\frac{1}{h}+\frac{1}{\hat{e}_1}}$ for all $i=2,...,\overline{N}+1$ and $Cov\left[s_i,s_j\left|s_1;\mathbf{e}_{-1},\hat{e}_1\right.\right]=\frac{1}{h}-\frac{\frac{1}{h^2}}{\frac{1}{h}+\frac{1}{\hat{e}_1}}$ for all $i,j=2,...,\overline{N}+1$ where $i\neq j$. Using this result, we have

$$\begin{aligned}
& V\left(\alpha^*,m_1,\mathbf{e}^*_{-1},\hat{e}_1\right)\\
= & -\left(\begin{array}{c}\sum_{i=2}^{\overline{N}+1}\left(\alpha^*-\hat{\alpha}_{-1}\right)^2 Var\left[s_i\left|s_1;\mathbf{e}^*_{-1},\hat{e}_1\right.\right]\\
+2\sum_{i=2}^{\overline{N}}\sum_{j=i+1}^{\overline{N}+1}\left(\alpha^*-\hat{\alpha}_{-1}\right)^2 Cov\left[s_i,s_j\left|s_1;\mathbf{e}^*_{-1},\hat{e}_1\right.\right]+Var\left[y\left|s_1;\mathbf{e}^*_{-1},\hat{e}_1\right.\right]\end{array}\right)\\
= & -\overline{N}\left(\alpha^*-\hat{\alpha}_{-1}\right)^2\left(\frac{1}{\hat{e}_1+h}+\frac{1}{e^*}\right)-2\left(\alpha^*-\hat{\alpha}_{-1}\right)^2\sum_{i=2}^{\overline{N}}\left(\overline{N}+1-i\right)\frac{1}{\hat{e}_1+h}-\frac{1}{h+\hat{e}_1+\overline{N}e^*}
\end{aligned}$$

Observe now that the first two terms are non-positive and strictly negative whenever $e_1\neq e^*$. On the other hand, e^* is precisely defined by

$$e^*=\arg\max_{e_1}\left\{-\frac{1}{h+e_1+\overline{N}e^*}-c\left(e_1\right)\right\}$$

This completes the proof. ■

Proof of Proposition 4. We prove here only the case of convex costs, where the best equilibrium has every agent acquire the same amount of information. The case of concave costs is trivial and therefore omitted.

When costs, are convex and every agent acquires the same amount of information, group utility can be written as

$$W\left(N+\delta-1\right)=\left(N+\delta\right)\left(-\frac{1}{h+\left(N+\delta\right)e\left(\delta\right)}-c\left(e\left(\delta\right)\right)\right)$$

where $\delta=0,1$, and $e\left(\delta\right)$ is defined by

$$\frac{1}{\left(h+\left(N+\delta\right)e\left(\delta\right)\right)^2}=c'\left(e\left(\delta\right)\right) \tag{39}$$

Per capita utility is defined as

$$\frac{W(N+\delta-1)}{(N+\delta)} = \left(-\frac{1}{h+(N+\delta)e(\delta)} - c(e(\delta))\right)$$

Differentiating condition (39) we obtain

$$\frac{de}{d\delta} = \frac{\frac{-2e(\delta)}{(h+(N+\delta)e(\delta))^3}}{\frac{2(N+\delta)}{(h+(N+\delta)e(\delta))^3} + c''(e(\delta))} \tag{40}$$

Differentiating per capita-welfare with respect to δ, we obtain

$$\frac{\partial\left(\frac{W(N+\delta-1)}{(N+\delta)}\right)}{\partial\delta} = \left(\frac{(N+\delta)}{(h+(N+\delta)e(\delta))^2} - c'(e(\delta))\right)\frac{de}{d\delta} + \frac{e(\delta)}{(h+(N+\delta)e(\delta))^2}$$

Substituting from (40) and using condition (39) we obtain

$$\begin{aligned}\frac{\partial\left(\frac{W(N+\delta-1)}{(N+\delta)}\right)}{\partial\delta} &= -\frac{(N+\delta-1)c'(e(\delta))\frac{2e(\delta)}{(h+(N+\delta)e(\delta))^3}}{\frac{2(N+\delta)}{(h+(N+\delta)e(\delta))^3} + c''(e(\delta))} + \frac{e(\delta)}{(h+(N+\delta)e(\delta))^2} \\ &= \frac{e(\delta)}{(h+(N+\delta)e(\delta))^2}\left(\frac{2c'(e(\delta)) + (h+(N+\delta)e(\delta))c''(e(\delta))}{2(N+\delta)c'(e(\delta)) + (h+(N+\delta)e(\delta))c''(e(\delta))}\right) > 0.\end{aligned}$$

The first-best value of per capita utility is increasing as well, with derivative $\frac{e(\delta)}{(h+(N+\delta)e(\delta))^2}$.

Consider now the limiting case as N tends to infinity. First, we show that equilibrium per capita utility goes to zero as N goes out of bounds. The reason is that $\lim_{N\to\infty} e(N) = 0$ and $\lim_{N\to\infty} Ne(N) = \infty$, so that the aggregate precision of information goes out of bounds while the individual precision levels (and thus the cost of acquiring these precisions) goes to zero. To see this, observe that $Ne(N)$ cannot converge to a finite number as $N \to \infty$. The first-order condition for any given N reads

$$\frac{1}{(h+Ne(N))^2} = c'(e(N))$$

$\lim_{N\to\infty} Ne(N) < \infty$. Then, since the left-hand side attains a finite value as N goes out of bounds, the right-hand side must attain the same value. So $e(N)$ would be bounded below by a positive constant. But then, $\lim_{N\to\infty} Ne(N) = \infty$ contradicting the initial hypotheses. Hence,

$$\lim_{N\to 0}\left(-\frac{1}{h+Ne(N)} - c(e(N))\right) = 0$$

Second, we show that first-best per capita utility goes to zero as N tends to infinity. The precision level in the first-best satisfies

$$\frac{N}{(h+Ne^*(N))^2} = c'(e^*(N))$$

Clearly again, $\lim_{N\to\infty} Ne^*(N) = \infty$. If the limit was bounded then $e^*(N)$ would grow in N, contradicting that $Ne^*(N)$ attained a fixed value. Multiply both sides by $e^*(N)$ and rearrange the equation to obtain

$$\frac{Ne^*(N)}{h+Ne^*(N)}\frac{1}{h+Ne^*(N)} = e^*(N)\,c'(e^*(N)).$$

Since $\frac{Ne^*(N)}{h+Ne^*(N)} \leq 1$, the left hand side goes to zero as N goes out of bounds. So the right-hand side must do the same. Since $ec'(e)$ is monotonic in e and equal to zero if and only if $e=0$, this implies that $\lim_{N\to\infty} e^*(N) = 0$. So, we have shown that

$$\lim_{N\to\infty}\left(-\frac{1}{h+Ne^*(N)} - c(e^*(N))\right) = 0.$$

Thus, per capita utility tends to its first-best value as N tends to infinity. ■

Proof of Proposition 7. Recall that $\hat{k} \equiv k+\gamma$. We can write (28) as

$$\frac{1}{\left(h+\left(\hat{k}+\left(K-\hat{k}\right)\left(\frac{K-\hat{k}}{N-\hat{k}}\right)^{\frac{1}{\eta-1}}\right)\overline{e}\right)^2} = c'(\overline{e}) \tag{41}$$

Notice that we can express the cost of acquiring information for the unimportant agents as

$$c\left(\left(\frac{K-k}{N-k}\right)^{\frac{1}{\eta-1}}\overline{e}\right) = \left(\frac{K-k}{N-k}\right)^{\frac{\eta}{\eta-1}} c(\overline{e}).$$

So we can express welfare as follows:

$$\begin{aligned} W\left(K,\hat{k}\right) &= -\frac{N}{h+\left(\hat{k}+\left(K-\hat{k}\right)\left(\frac{K-\hat{k}}{N-\hat{k}}\right)^{\frac{1}{\eta-1}}\right)\overline{e}} - \hat{k}c\left(\overline{e}\right) - \left(N-\hat{k}\right)\left(\frac{K-\hat{k}}{N-\hat{k}}\right)^{\frac{\eta}{\eta-1}} c\left(\overline{e}\right) \\ &= -\frac{N}{h+\left(\hat{k}+\left(K-\hat{k}\right)\left(\frac{K-\hat{k}}{N-\hat{k}}\right)^{\frac{1}{\eta-1}}\right)\overline{e}} - \left(\hat{k}+\left(K-\hat{k}\right)\left(\frac{K-\hat{k}}{N-\hat{k}}\right)^{\frac{1}{\eta-1}}\right) c\left(\overline{e}\right) \end{aligned}$$

Differentiating with respect to $\hat{k}$ and using (41) to simplify the resulting expression we obtain

$$\begin{aligned} \frac{\partial W\left(K,\hat{k}\right)}{\partial \hat{k}} &= \left(\hat{k}+\left(K-\hat{k}\right)\left(\frac{K-\hat{k}}{N-\hat{k}}\right)^{\frac{1}{\eta-1}}\right)(N-1)\,c'\left(\overline{e}\right)\frac{\partial \overline{e}}{\partial \hat{k}} \\ &\quad + \left(N\overline{e}c'\left(\overline{e}\right)-c\left(\overline{e}\right)\right)\frac{\partial}{\partial \hat{k}}\left(\hat{k}+\left(K-\hat{k}\right)\left(\frac{K-\hat{k}}{N-\hat{k}}\right)^{\frac{1}{\eta-1}}\right) \end{aligned}$$

From (41), noting that $\left(h+\left(\hat{k}+\left(K-\hat{k}\right)\left(\frac{K-\hat{k}}{N-\hat{k}}\right)^{\frac{1}{\eta-1}}\right)\overline{e}\right)^3 = \left(c'\left(\overline{e}\right)\right)^{-\frac{3}{2}}$, we find

$$\frac{d\overline{e}}{d\hat{k}} = -\frac{2\overline{e}\frac{\partial}{\partial \hat{k}}\left(\hat{k}+\left(K-\hat{k}\right)\left(\frac{K-\hat{k}}{N-\hat{k}}\right)^{\frac{1}{\eta-1}}\right)}{2\left(\hat{k}+\left(K-\hat{k}\right)\left(\frac{K-\hat{k}}{N-\hat{k}}\right)^{\frac{1}{\eta-1}}\right)+\left(c'\left(\overline{e}\right)\right)^{-\frac{3}{2}}c''\left(\overline{e}\right)}$$

Substituting back into the welfare function, we obtain

$$\begin{aligned} \frac{\partial W\left(\hat{k}\right)}{\partial \hat{k}} &= -(N-1)\,c'\left(\overline{e}\right)\overline{e}\frac{2\left(\hat{k}+\left(K-\hat{k}\right)\left(\frac{K-\hat{k}}{N-\hat{k}}\right)^{\frac{1}{\eta-1}}\right)}{2\left(\hat{k}+\left(K-\hat{k}\right)\left(\frac{K-\hat{k}}{N-\hat{k}}\right)^{\frac{1}{\eta-1}}\right)+\left(c'\left(\overline{e}\right)\right)^{-\frac{3}{2}}c''\left(\overline{e}\right)}\frac{\partial}{\partial \hat{k}}\left(\hat{k}+\left(K-\hat{k}\right)\left(\frac{K-\hat{k}}{N-\hat{k}}\right)^{\frac{1}{\eta-1}}\right) \\ &\quad + \left(N\overline{e}c'\left(\overline{e}\right)-c\left(\overline{e}\right)\right)\frac{\partial}{\partial \hat{k}}\left(\hat{k}+\left(K-\hat{k}\right)\left(\frac{K-\hat{k}}{N-\hat{k}}\right)^{\frac{1}{\eta-1}}\right) \\ &= \frac{\partial}{\partial \hat{k}}\left(\hat{k}+\left(K-\hat{k}\right)\left(\frac{K-\hat{k}}{N-\hat{k}}\right)^{\frac{1}{\eta-1}}\right)\times \\ &\quad \left(N\overline{e}c'\left(\overline{e}\right)-c\left(\overline{e}\right)-(N-1)\,c'\left(\overline{e}\right)\overline{e}\frac{2\left(\hat{k}+\left(K-\hat{k}\right)\left(\frac{K-\hat{k}}{N-\hat{k}}\right)^{\frac{1}{\eta-1}}\right)}{2\left(\hat{k}+\left(K-\hat{k}\right)\left(\frac{K-\hat{k}}{N-\hat{k}}\right)^{\frac{1}{\eta-1}}\right)+\left(c'\left(\overline{e}\right)\right)^{-\frac{3}{2}}c''\left(\overline{e}\right)}\right) \end{aligned}$$

Using the fact that $c\left(\overline{e}\right)=\frac{\overline{e}c'(\overline{e})}{\eta}$, we obtain

$$\frac{\partial W\left(K,\hat{k}\right)}{\partial\hat{k}} = \frac{\partial}{\partial\hat{k}}\left(\hat{k}+\left(K-\hat{k}\right)\left(\frac{K-\hat{k}}{N-\hat{k}}\right)^{\frac{1}{\eta-1}}\right)\overline{e}c'\left(\overline{e}\right)\times$$
$$\left(N-\frac{1}{\eta}-(N-1)\frac{2\left(\hat{k}+\left(K-\hat{k}\right)\left(\frac{K-\hat{k}}{N-\hat{k}}\right)^{\frac{1}{\eta-1}}\right)}{2\left(\hat{k}+\left(K-\hat{k}\right)\left(\frac{K-\hat{k}}{N-\hat{k}}\right)^{\frac{1}{\eta-1}}\right)+\left(c'\left(\overline{e}\right)\right)^{-\frac{3}{2}}c''\left(\overline{e}\right)}\right)$$

Simplifying, the change in welfare can be written as

$$\frac{\partial W\left(K,\hat{k}\right)}{\partial\hat{k}} = \frac{\partial}{\partial\hat{k}}\left(\hat{k}+\left(K-\hat{k}\right)\left(\frac{K-\hat{k}}{N-\hat{k}}\right)^{\frac{1}{\eta-1}}\right)\times$$
$$\overline{e}c'\left(\overline{e}\right)\frac{\left(1-\frac{1}{\eta}\right)2\left(\hat{k}+\left(K-\hat{k}\right)\left(\frac{K-\hat{k}}{N-\hat{k}}\right)^{\frac{1}{\eta-1}}\right)+\left(N-\frac{1}{\eta}\right)\frac{c''(\overline{e})}{(c'(\overline{e}))^{\frac{3}{2}}}}{2\left(\hat{k}+\left(K-\hat{k}\right)\left(\frac{K-\hat{k}}{N-\hat{k}}\right)^{\frac{1}{\eta-1}}\right)+\frac{c''(\overline{e})}{(c'(\overline{e}))^{\frac{3}{2}}}}$$

Since the expression in the second line is unambiguously positive, the sign of $\frac{\partial W\left(K,\hat{k}\right)}{\partial\hat{k}}$ is given by the sign of $\frac{\partial}{\partial\hat{k}}\left(\hat{k}+\left(K-\hat{k}\right)\left(\frac{K-\hat{k}}{N-\hat{k}}\right)^{\frac{1}{\eta-1}}\right)$. It suffices to show that this expression is positive for all $\hat{k}$. Straightforward differentiation and some manipulations give

$$\frac{\partial}{\partial\hat{k}}\left(\hat{k}+\left(K-\hat{k}\right)\left(\frac{K-\hat{k}}{N-\hat{k}}\right)^{\frac{1}{\eta-1}}\right)=1-\left(\frac{K-\hat{k}}{N-\hat{k}}\right)^{\frac{1}{\eta-1}}\frac{\eta\left(N-\hat{k}\right)-\left(K-\hat{k}\right)}{(\eta-1)\left(N-\hat{k}\right)}$$

The expression is positive iff

$$1-\left(\frac{K-\hat{k}}{N-\hat{k}}\right)^{\frac{1}{\eta-1}}\frac{\eta\left(N-\hat{k}\right)-\left(K-\hat{k}\right)}{(\eta-1)\left(N-\hat{k}\right)}>0$$

or equivalently, iff

$$1>\frac{1}{\eta-1}\left(\frac{K-\hat{k}}{N-\hat{k}}\right)^{\frac{1}{\eta-1}}\left(\eta-\frac{K-\hat{k}}{N-\hat{k}}\right)$$

To see that this condition is always satisfied, let $\tau\equiv\frac{K-\hat{k}}{N-\hat{k}}$. Observe that $0\leq\tau<1$ for all $\hat{k},N$ and all $K<N$. Rearrange the inequality as

$$\eta-1>\tau^{\frac{1}{\eta-1}}\left(\eta-\tau\right)$$

Observe that for $\tau = 1$, the the right-hand side takes value $\eta - 1$. Moreover, $\tau^{\frac{1}{\eta-1}} (\eta - \tau)$ is an increasing function of τ, since

$$\frac{\partial}{\partial \tau}\left(\tau^{\frac{1}{\eta-1}} (\eta - \tau)\right) = \tau^{\frac{1}{\eta-1}} \left(\frac{\eta - \tau - (\eta - 1)\tau}{(\eta - 1)\tau}\right) = \tau^{\frac{1}{\eta-1}} \left(\frac{\eta(1-\tau)}{(\eta - 1)\tau}\right) > 0.$$

So, we have shown that welfare is monotone increasing in the number of experts. ■

Proof of Proposition 9. Using the equilibrium condition for the precisions, (34) and the specific cost functions we find that

$$e_j = \pi^{\frac{1}{\eta-1}} \left(\frac{z_i}{z_j}\right)^{\frac{1}{\eta-1}} e_i$$

Using this result, we can compute the effective aggregate precision of information as

$$e_i + \pi \sum_{j \neq i} e_j = e_i \left(1 + \pi^{\frac{\eta}{\eta-1}} \sum_{j \neq i} \left(\frac{z_i}{z_j}\right)^{\frac{1}{\eta-1}}\right) \tag{42}$$

Substituting into the president's first-order condition we obtain

$$\left(e_i \left(1 + \pi^{\frac{\eta}{\eta-1}} \sum_{j \neq i} \left(\frac{z_i}{z_j}\right)^{\frac{1}{\eta-1}}\right)\right)^{-2} = z_i e_i^{\eta-1} \tag{43}$$

We can solve this condition for the president's equilibrium precision of information

$$e_i^* = z_i^{\frac{-1}{\eta+1}} \left(1 + \pi^{\frac{\eta}{\eta-1}} \sum_{j \neq i} \left(\frac{z_i}{z_j}\right)^{\frac{1}{\eta-1}}\right)^{\frac{-2}{\eta+1}}$$

Thus, (42) can be written as

$$e_i^* + \pi \sum_{j \neq i} e_j^* = z_i^{\frac{-1}{\eta+1}} \left(1 + \pi^{\frac{\eta}{\eta-1}} \sum_{j \neq i} \left(\frac{z_i}{z_j}\right)^{\frac{1}{\eta-1}}\right)^{\frac{\eta-1}{\eta+1}}$$

On the other hand, the cost of information acquisition is

$$\frac{1}{\eta}\sum z_j\frac{e_j^{*\eta}}{\eta} = \frac{z_i}{\eta}e_i^{*\eta}+\frac{1}{\eta}\sum_{j\neq i}z_j\pi^{\frac{\eta}{\eta-1}}\left(\frac{z_i}{z_j}\right)^{\frac{\eta}{\eta-1}}e_i^{*\eta}=\frac{1}{\eta}z_ie_i^{*\eta}\left(1+\pi^{\frac{\eta}{\eta-1}}\sum_{j\neq i}\left(\frac{z_i}{z_j}\right)^{\frac{1}{\eta-1}}\right)$$

$$= \frac{1}{\eta}z_i^{\frac{-\eta+\eta+1}{\eta+1}}\left(1+\pi^{\frac{\eta}{\eta-1}}\sum_{j\neq i}\left(\frac{z_i}{z_j}\right)^{\frac{1}{\eta-1}}\right)^{\frac{-2\eta+\eta+1}{\eta+1}}=\frac{1}{\eta}z_i^{\frac{1}{\eta+1}}\left(1+\pi^{\frac{\eta}{\eta-1}}\sum_{j\neq i}\left(\frac{z_i}{z_j}\right)^{\frac{1}{\eta-1}}\right)^{\frac{1-\eta}{\eta+1}}$$

Let $W(i)$ denote welfare as a function of the identity of the president. Using the developments above, we have

$$W(i)=-\left[1+\frac{1}{\eta}\right]z_i^{\frac{1}{\eta+1}}\left(1+\pi^{\frac{\eta}{\eta-1}}\sum_{j\neq i}\left(\frac{z_i}{z_j}\right)^{\frac{1}{\eta-1}}\right)^{\frac{1-\eta}{\eta+1}}$$

Noting that $W(i)$ is maximal if $z_i=\min_j z_j$ completes the proof of the proposition. ■

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